ergy, kinetic and potential energy, work and heat, and so forth. Most disturbingly, we mix up energy with the fundamental quantities flowing in physical processes, namely, electricity, heat, and motion.

Actually we have to learn very little about energy and what we learn repeats itself again and again in every field of physics. From what we have discussed so far, we recognize that there is just a single quantity called *energy* which accompanies all processes. This quantity has the following properties:

- Energy is released and used in processes.
- Energy can be transported from system to system.
- Energy can be stored in systems.
- Energy is conserved; it can neither be created nor destroyed.

The second and third items in the list make energy a quantity to which the laws of accounting can be applied; in other words, energy satisfies a *law of balance* (Section 2.4). The properties of energy will now be investigated more carefully and with quantitative means.

2.2 POWER: THE RATE AT WHICH ENERGY IS RELEASED IN A PROCESS

Nature presents us with a perfectly simple process which can serve as the archetype of physical processes—a *waterfall*. Other processes are interpreted analogously: a process consists of the flow a fluidlike quantity from a higher to a lower level (Fig. 2.3). We introduce *energy* as the measure of how much the fluidlike quantity is working, i.e., how much it can achieve, when it falls down a gradient of its potential. *Power* is the rate at which the fluidlike quantity is working. We will say that energy is released in a process, and power—the rate of working—is the rate at which energy is released. Common sense reasoning indicates that the power of a process will depend on the flow of the fluidlike quantity and the height of its fall.[2]

2.2.1 Power of an Electric Process

A simple experiment which can be used to quantify the measure of power is the electric heating of water. The rate of heating of water may be measured in terms of the rate of change of its temperature. If we always take the same amount of water at the same temperature, and observe the same rate of change of temperature, we can be sure that the electric process is "working at the same rate." In terms of energy we may say that this quantity has been released at the same rate in the immersion heater every time we repeat the experiment. On the other hand, if twice as much water can be heated at this rate, the electric processes must run at twice the rate.

Different runs of this experiment show that the rate of change of temperature is the same for identical bodies of water whenever the product of electric current through the

2. This is how Sadi Carnot expressed his idea of the power of heat. See the Introduction for a short discussion of his idea and the roots of common sense conceptualizations of phenomena and processes.

Electric Process

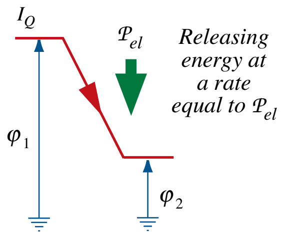

Figure 2.8: Energy released in an electric process. The rate at which energy is released (the power of the electric process) depends upon the flow of charge and the potential difference.

immersion heater and the electric potential difference across the device is the same. In other words, the rate of working of the electric process can be measured in terms of the product of electric current and voltage:

$$\mathcal{P}_{el} = -\Delta\varphi_{el} I_Q \quad \text{or} \quad \mathcal{P}_{el} = U I_Q \tag{2.1}$$

The symbol $\mathcal{P}$ is used for the rate at which energy is released; from now on we will call this quantity *power*. Therefore, we speak of the *electric power of a flow of electric charge*. The SI unit of power is the Watt (W). The minus sign in the first form of the equation is arbitrary. It means that the power of a voluntary process is counted as a positive number, while the power of an involuntary process is taken to be negative.

The equation can be interpreted graphically using the waterfall image of a process (Fig. 2.8). In an electric process that drives another process, electric charge flows "downhill" through a potential difference and in turn releases energy at a rate that depends upon the flow of charge and the potential difference in the simple manner indicated by Equ.(2.1).

2.2.2 Hydroelectric Power Plants and the Power of a Gravitational Process

We need a measure of the *power of a fall of water*, i.e., the rate at which energy is released in a gravitational process. By allowing water to accomplish a measurable result at a certain rate, we can define the power of a fall of water. Data on hydroelectric power plants yields the information we need (Table 2.2). If we take the product of electric current and voltage at the terminal of the generator as the measure of the rate of working of the water rushing down from the artificial lake to the turbine, we can see which factors determine the rate at which a waterfall releases energy.

Table 2.2: Examples of hydraulic power plants [a]

Hydraulic power plant	Current of Mass I_m / kg/s	Vertical fall of water Δh / m	Voltage and current [b] UI_Q / V · A	$UI_Q / I_m \Delta h$
Bavona	18,000	890	$137 \cdot 10^6$	8.6
Nendaz	45,000	1014	$384 \cdot 10^6$	8.4
Handeck III	12,500	445	$48 \cdot 10^6$	8.6
Chatelard	16,000	814	$107 \cdot 10^6$	8.2
Tiefencastel	16,700	374	$50 \cdot 10^6$	8.0

a. Hydraulic power plants with artificial lakes in Switzerland.
b. Product of voltage and electric current measured for the generator.

The results in Table 2.2 demonstrate that—except for an almost constant factor—the current of mass of water (measured in cubic meters per second) and the vertical drop of the water from the artificial lake to the turbine and generator station (measured in meters) determine the rate at which energy is released by the falling water. In fact, this quantity depends linearly on both factors. Doubling the current of water, or doubling the drop, will each lead to a doubling of the rate of release of energy.

Power of a waterfall. Specifying a waterfall first of all means quantifying the flow of water falling down. This is done with the help of the *current of (gravitational) mass* I_m (measured in kilograms per second). The second obvious quantity determining the properties of a waterfall is the vertical drop Δh (measured in meters).

The power of a waterfall, i.e., gravitational power, depends upon another parameter which is suggested by the fact that the strength of the gravitational field g must play a role. We expect the drop of water through a certain height to accomplish much less on the surface of the Moon than on the surface of our planet. Now we are ready to calculate the rate at which energy is released:

$$\left|\mathcal{P}_{grav}\right| = \left|g \Delta h I_m\right| \tag{2.2}$$

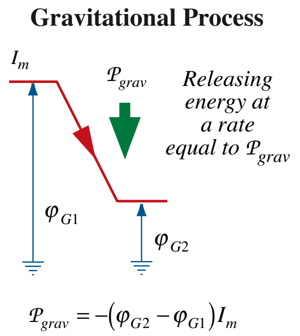

Figure 2.9: Energy released in a gravitational process. The rate at which energy is released (the power of the gravitational process) depends upon the flow of mass and the gravitational potential difference.

Potential. There is a simple graphical interpretation of the formula for the power of a waterfall (Fig. 2.9). We combine the first two factors on the right side of Equ.(2.2) into a new quantity which we call the *level* or *potential of gravitational processes*:

$$\varphi_G = g h \tag{2.3}$$

According to the results in Table 2.2, g should be somewhat larger than 8 N/kg. We know from independent measurements that it is closer to 9.8 N/kg (Section 1.4.4). The discrepancy is a result of the imperfection of the processes in power plants.

We may now write the power of the process as the product of the difference of the gravitational potential and the current of mass falling through this difference of levels:

$$\mathcal{P}_{grav} = -\Delta \varphi_G I_m \tag{2.4}$$

Note the analogy between this result and the one for electricity (Equ.(2.1)). The expression introduced for the gravitational potential is analogous to the one found in Chapter 1 (see Fig. 1.26).

2.2.3 The Efficiency of Processes

Note that the experimental determination of the factor in the last column of Table 2.2 leads to values that are a little bit smaller than g. This is due to the fact that the processes leading from the waterfall to the generator are not ideal: some of the energy released by the water is used for other purposes—mostly for the production of heat as a result of friction.

Ideally, all the energy released in a process would be used for the desired follow up process. Realistically, this does not happen, since parallel processes such as friction bind part of the energy released (Fig. 2.10). To measure the efficiency of the transfer of energy to the desired process, the ratio of the powers involved is used:

$$efficiency = \frac{\mathcal{P}_{desired\ process}}{\mathcal{P}_{driving\ process}} \tag{2.5}$$

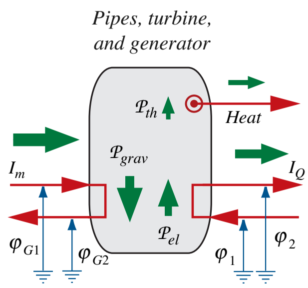

Figure 2.10: In a non-ideal coupling of processes, less than 100% of the energy released is used in the desired process.

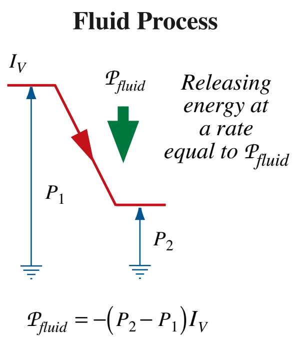

Figure 2.11: Energy released in a fluid process (hydraulic process). The rate at which energy is released (the power of the fluid process) depends upon the flow of volume of fluid and the pressure difference.

The power of an electric process is measured as the product of voltage and electric current (Equ.(2.1)). Applying this rule to the values presented in Table 2.2 we see that the overall efficiencies of modern hydroelectric power plants are quite high, of the order of 80% to 90%.

2.2.4 Power of a Hydraulic Process

Analogical reasoning suggests, and experiments confirm, that the type of relation found for the power of electric and gravitational potentials also holds for hydraulic processes:

$$\mathcal{P}_{hyd} = -\Delta P I_V \qquad \textbf{(2.6)}$$

Just consider a turbine driving an electric generator. The electric process is found to be identical as long as the product of the pressure difference and the flux of volume is kept constant. In summary, all types of processes investigated demonstrate the same basic structure (see Section 2.2.6 and Table 2.3): knowing one field of nature helps us to understand other subjects.

2.2.5 Power in Inductive Processes

So far, we have studied devices such as pumps, turbines and generators, artificial lakes and pipes, resistors, electric engines, etc. They all demonstrate that the release of energy is followed by its use when processes are coupled.

Inductive elements (Section 1.6) seem to confront us with a somewhat different case. First, the other devices work strictly in one way—in resistors, volume or charge always flow "downhill"—while processes in inductors run both ways. Second, most of the systems mentioned before can run in steady state without involving the storage of energy; inductive devices, however, work dynamically only, and they also serve as energy storage devices.

Third and most important, it is not readily apparent if there are two processes coupled in such devices, one running "downhill", driving the second one "uphill." Closer inspection shows, however, that there are processes coupled to the obviously visible electric or hydraulic ones. Let us see what they are in the case of electromagnetic induction.

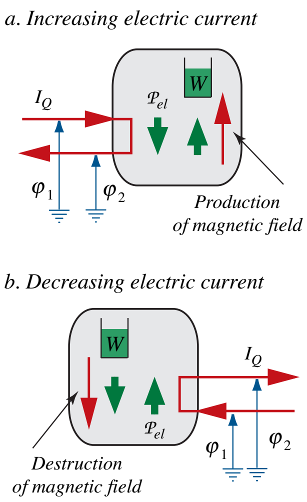

Figure 2.12: In an inductive electric process, energy is released or bound. The process is coupled to the creation or destruction of a magnetic field which acts as the storage device for energy in the inductive element.

The phenomenon of electromagnetic induction is coupled to the growth or decay of magnetic fields due to electric currents. The magnetic field acts as the storage system for the energy (Section 2.4) which is released or bound by the electric current—depending on whether the current is increasing in time, or decreasing. If the electric current through an inductive device is increasing with time, i.e. if $dI_Q/dt > 0$, it runs "downhill" through the inductive potential difference $\Delta\varphi_L$ (Section 1.6). We have just learned that this process is associated with the release of energy at the rate

$$\mathcal{P}_{el} = -\Delta\varphi_L I_Q \qquad \textbf{(2.7)}$$

There should be a process running "uphill" on the energy made available. This process exists: it is the building up of a magnetic field which at the same time acts as the storage device for the energy released in the electric process (Fig. 2.12a).

If the electric current through the inductive element decreases with time, i.e. if $dI_Q/dt < 0$, the magnetic field decreases as well, releasing energy which is picked up by the electric current. As a result, this current is driven "uphill" through the induced potential difference $\Delta\varphi_L$ (Fig. 2.12b).

The case of hydraulic induction is quite analogous. However, here we do not have a magnetic field associated with the current. Rather it is the quantity of motion of the flowing fluid which is built up or reduced in the device which acts similarly to the magnetic field.

2.2.6 Processes and Power in General

If a fluidlike quantity falls "downhill" it releases energy at a certain rate. This rate we call the *power* of the process. The energy that is released drives a follow-up process "uphill", and it is said to be used by or bound to the flowing quantity (Fig. 2.13). The law for the energy released or used is this:

The power of a process always depends on two factors—the potential difference and the current flowing through this potential difference:

$$\mathcal{P}_X = -\Delta\varphi_X I_X \tag{2.8}$$

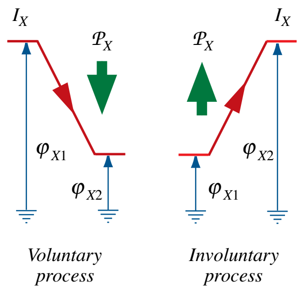

Figure 2.13: Processes and the power of processes. The same fundamental structure is discovered in all physical processes.

The letter X stands for the flowing fluidlike quantity which determines the type of process: mass, volume, and electric charge for gravitational, hydraulic, and electric processes, respectively (Table 2.3). For a given process, we have to use the proper fluidlike quantity and its associated potential. Thus, for a hydraulic process, X corresponds to V, and $\Delta\varphi_X$ corresponds to ΔP.

Table 2.3: Comparison of different processes

	Flowing quantity	Current	Potential	Potential difference	Power
Gravity	Gravitational mass	Current of gravitational mass	Gravitational potential	$\Delta\varphi_G$	$-\Delta\varphi_G I_m$
Hydraulics	Volume of fluid	Current of volume	Pressure	ΔP	$-\Delta P I_V$
Electricity	Electric charge	Current of electric charge	Electric potential	$\Delta\varphi_{el} = -U$	$-\Delta\varphi_{el} I_Q = U I_Q$

Amounts of energy released or used in a process. Sometimes, we want to be able to say "how much has happened" in a process. In other words, we want to know how much energy has been released or bound as the result of a process lasting for a certain period. The amount of energy released in a process—which is sometimes called *work*[3]—can be obtained by integrating the power over time (Fig. 2.14). In general, this quantity can also be calculated as the product of the amount X_e of the fluidlike quantity flowing through a potential difference, and the potential difference $\Delta\varphi$:

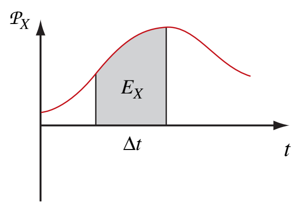

Figure 2.14: The integral over time of the power of a process yields the energy released or used in that process.

$$W_X = -\Delta\varphi_X X_e \qquad (2.9)$$

This expression is correct only if the potential difference stays constant during the process. The relation is particularly simple to prove for a process running at a constant rate. The unit of energy (released) is the Joule (1 J = 1 W·s).

2.2.7 Electric and Hydraulic Circuits: The Balance of Power

An indication of the balance of energy comes from the consideration of energy released or bound in closed electric and hydraulic circuits: the sum of all terms of electric or hydraulic power add up to zero.

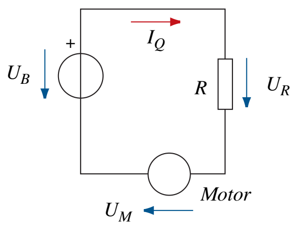

Figure 2.15: The sum of the potential differences in a closed circuit is always zero. Therefore, the sum of the electric power terms of all the elements combined must be zero as well.

This is a consequence of Kirchhoff's second rule which we encountered in hydraulics and electricity (Chapter 1). Consider a simple electric circuit containing a battery, a resistor, and an electric motor (Fig. 2.15). The current of charge flowing through all three elements is the same, and the voltages across them add up to zero:

$$U_B + U_R + U_M = 0 \qquad (2.10)$$

The current is flowing through each of the elements leading to the release or binding of energy. If we multiply Equ.(2.10) by the current I_Q, we obtain $U_B I_Q + U_R I_Q + U_M I_Q = 0$. Since the terms represent the electric power in the elements, this is equivalent to

$$\mathcal{P}_{el,B} + \mathcal{P}_{el,R} + \mathcal{P}_{el,M} = 0 \qquad (2.11)$$

This means that the energy bound in the electric process in the battery is equal to the energy released in the resistor and the motor combined as the consequence of the fall of the electric charge. In everyday language we say that the energy delivered by the battery is used by the resistor and the motor.

2.3 ENERGY TRANSFER AND ENERGY CARRIERS

Energy released in a process does not come out of the blue, and energy that is bound does not disappear. Either it is transferred into or out of the system or it comes from storage or will be stored (Section 2.4). Here we shall investigate the transfer of energy. There is a simple form of coupling of the flow of the fluidlike quantities with the flow of energy into and out of systems. It is as if mass, volume, and charge acted as *carriers of energy* in the processes they are responsible for.

2.3.1 Energy Carriers, Potentials, and Energy Currents

A simple example demonstrates how nature works. Consider the steady-state flow of a viscous fluid through a straight pipe as in Fig. 2.2. So far we have introduced the concept of energy in the following manner: since the fluid flows from a point of high

3. The words *power* and *work* are used inconsistently in different fields of physics. In mechanics, for example, work means a quantity of energy *transferred*, not released.

pressure to a point of lower pressure, energy is released in the hydraulic process at a certain rate (Fig. 2.16). The energy released is bound in the following thermal process. Remember that the production of heat due to friction is all that happens in the pipe; therefore, we assume that 100% of the energy released is bound in the follow up process.

To be specific, let us introduce concrete numbers. Assume there is a fluid current of 0.10 m^3/s, and a pressure drop of 0.50 bar. According to Equ.(2.6), energy must be released at a rate of 5.0 kW. In other words, 5000 J energy are released each second and made available for the production of heat. We believe that the energy must be supplied to the system. Since the only possibility for this to happen is through the flow of fluid into—and out of—the pipe, we say that the fluid flowing under pressure carries with it some energy: we associate an *energy current* with the fluid (Fig. 2.16). In this sense we can call the fluid the *energy carrier* with respect to the system.

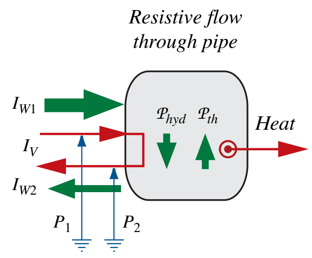

Figure 2.16: The energy released in the "fall" of fluid from high to low pressure must be supplied to the system. It is flowing into—and out of—the device with the fluid under pressure.

Naturally, we should expect the energy current to depend upon two factors. First, it must be proportional to the current of fluid; two equal currents under identical conditions will have twice the effect of a single one. Second, the pressure of the fluid must play a role. Let us see how energy and carrier currents are related.

If a fluid flowing into the system at a certain level (pressure) carries energy, so must the fluid flowing out of the system. Therefore, we assume that the rate at which energy is released is the difference between the currents of energy into and out of the pipe due to fluid flow. Since this makes the difference of the energy flows equal to the product of the pressure difference and the volume current, i.e.,

$$|I_{E1}| - |I_{E2}| = |(P_2 - P_1) I_V|$$

the simplest expression for a single current of energy I_E is the product of the flux of volume and the pressure of the fluid as it enters—or leaves—the system:

$$I_{E,fluid} = P I_V \qquad \textbf{(2.12)}$$

There is a simple image which can be used to remember this relation. We may look upon the pressure as the "load factor" of the "carrier current." The current of volume is "loaded" with energy according to the value of the pressure. The flux of energy therefore is the product of a carrier current and its load factor.

Again, this is the structure of energy flow in all fields of physics. Consider the different devices and processes studied so far—gravitational and electric ones in addition to hydraulic: we always arrive at exactly the same relation for the expected energy currents.

The flux of a current of energy entering—or leaving—a system is the product of the flux of the carrier current and its associated potential (Fig. 2.17):

$$I_{E,X} = \varphi_X I_X \qquad \textbf{(2.13)}$$

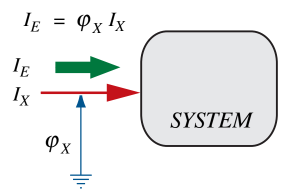

Figure 2.17: The relation of flux of energy, "carrier" flux, and the "load factor," represented in a process diagram.

As we have seen in Chapter 1, the electric potential is not an absolute quantity. Values of electric potentials must always be measured with respect to a chosen level, i.e., the "ground." The same is true for the gravitational potential; here on our planet we commonly measure levels or heights relative to sea level. Of the levels we know so far, only the hydraulic one is absolute. Fluxes of energy in electric and gravitational processes therefore do not have quite the same independent meaning as in fluid flow. Only the difference of two energy currents flowing into and out of a system together with a single current of a fluidlike quantity is independent of arbitrarily chosen levels. This difference is equal to the power of the associated process (Fig. 2.16).

This already tells us that the notion of energy being “carried” by the current of a fluidlike quantity should not bee taken too literally. In particular, as we shall see below, “carried” does not mean that the carrier current “contains” the energy being supplied. We should look upon Equ.(2.13) as meaning that energy always flows *at the same time* as the fluidlike quantity—rather than together with or directly bound to the carrier. It is certainly correct to state that *energy never flows alone*: at the same time, there must always be one or more flows of other physical quantities.[4]

2.3.2 Energy Transfer in Compression

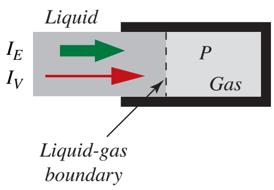

Figure 2.18: A gas in a vessel is compressed by a liquid flowing in. The gas has a pressure *P*.

There is an example of energy transfer that will play a particularly important role in our study of thermodynamics: energy flows associated with compression or expansion of a (compressible) fluid.

Imagine an imaginary wall separating a gas inside a container from a liquid that flows in or out so the gas is either compressed or expanded (Fig. 2.18). At the liquid-gas boundary we have a flux of volume of liquid at pressure P (which is the pressure of the gas enclosed by the liquid and the walls of the vessel). The current of volume of the liquid is I_V, so there is an energy flux $I_E = PI_V$ entering the gas. At the same time, the gas is compressed at a rate that equals the flow of volume of liquid. Since the volume of the gas is decreasing—we might say, volume of gas is “disappearing”—we describe the effect by a (negative) *production rate* of volume Π_V. In summary, a gas at pressure P being compressed at rate Π_V receives energy at the rate equal to

$$I_{E,comp} = -P\,\Pi_V \qquad (2.14)$$

2.4 ENERGY STORAGE AND THE BALANCE OF ENERGY

In some sense energy is like amounts of water: we can account for it. We have seen this principle applied in the steady state processes investigated in the previous sections. Energy flows through chains of processes, and since we believe that it is a conserved quantity, we know that the flow does not change in magnitude.

4. There is another point that needs to be taken into consideration. When we get into details of transport processes in later chapters, we shall see that there are three fundamentally different types of flows: conductive (flow through matter, caused by a potential gradient), convective (transport of a quantity stored in a fluid, as a consequence of fluid flow), and radiative (transport of a physical quantity with radiation). The relation between energy fluxes and fluxes of fluidlike quantities only holds for conductive transports. Conductive currents are those that are associated with their (own) potentials, so Equ.(2.13) (or Equ.(2.8), for that matter) make sense in this respect. A conductive current I_X is a current associated with or driven by the potential difference φ_X. As we shall see in Chapter 7, energy transfers in convection and radiation take different forms.

Changes in the flows in the course of time are possible, however, if energy is stored in systems. Only if we take into account storage of this quantity do we arrive at a general law of balance.

2.4.1 The General Law of Balance of Energy

Unless we believe that energy is either generated or disappears if chains of processes are interrupted, we must accept the idea that energy can be stored (Section 2.1). Bodies—and physical systems in general—can contain energy, and they can absorb it and emit it, thus changing the amounts stored.

As in the case of amounts of water—or amounts of electric charge—a law of balance relates what happens to the quantity stored as the consequence of flow into and out of the system. Because energy can neither disappear nor appear out of the blue, we know that amounts stored can only be changed as the result of flows. This is what we call the *law of balance of energy* for a system:

> *Energy can be stored and it can flow. The sum of all fluxes of energy with respect to a system tell us how fast the amount of energy stored will change:*

$$\frac{dE}{dt} = I_{E,net} \tag{2.15}$$

> *(Fig. 2.19). This form holds for every moment. For a process lasting for a certain period, we may also say that the change of the amount of energy stored is determined by the total amount of energy transferred into or out of the system:*

$$\Delta E = E_{e,net} \tag{2.16}$$

E_e is called an amount of energy exchanged as the result of a process. Note that one of the properties of energy—namely that it can be released and bound—does not appear in a law of balance. Releasing and binding take place inside the system being considered whereas a law of balance only speaks of the relation between amounts stored and amounts flowing into and out of the system.

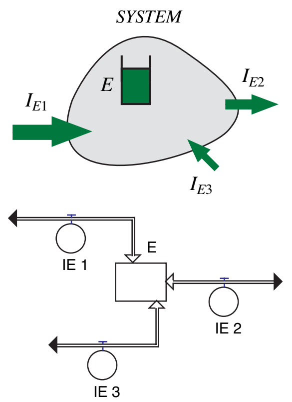

Figure 2.19: The law of balance of energy resembles the law of balance of amounts of water. The energy content of a system can only be changed as the result of flows of energy into and out of the system. Bottom: Graphical representation of law of balance in an SD diagram.

2.4.2 Storing Energy with the Help of Gravity

We know how to calculate energy transfers. If we add to this the knowledge contained in the law of balance, we can determine changes of quantities of energy contained in particular systems. A particularly useful and graphically intuitive example is the storage of liquids in containers in the gravitational field. If we fill a tank with water, we add energy to the system along with the fluid, and this energy can usually be regained if the water is let flow out.

Imagine a storage device such as an artificial lake having a certain shape. Water contained in it can flow down to a power station which is located at a certain level H below the bottom of the lake (Fig. 2.20). If we imagine a small amount of water having mass Δm at level h lowered to $h = 0$, the quantity of energy flowing out of the system is equal to $E_e = gh\Delta m$. This quantity is different for different layers of water in the lake. It is

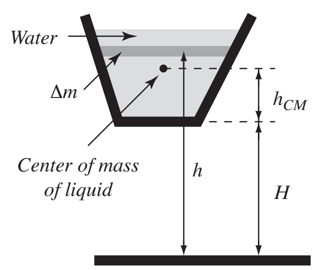

Figure 2.20: Water stored in an artificial lake contains a certain amount of energy relative to an arbitrary zero level.

quite intuitive that, on average, all the water is lowered from the level of the center of mass of the liquid to the bottom. In other words, the water comes from an average level $H + h_{CM}$. The total energy that flows out, and therefore the change of the energy of the storage system, equals

$$\Delta E = g\left(H + h_{CM}\right)m \tag{2.17}$$

There is a special form of this for a straight walled tank sitting at level $h = 0$ and being filled to level h_0. With $H = 0$, $h_{CM} = h_0/2$, and $m = \rho A h_0$, Equ.(2.17) becomes

$$\Delta E = g\left(0 + \frac{1}{2}h_0\right)\rho A h_0 = \frac{1}{2}\rho g A h_0^2$$

Here, A is the cross sectional area of the tank. If we introduce a gravitational capacitance of the storage device:

$$C_G = \frac{\Delta m}{\Delta\left(gh\right)} = \frac{\Delta\left(\rho A h\right)}{\Delta\left(gh\right)} = \frac{\rho A}{g}$$

the former expression can be converted to

$$\Delta E = \frac{1}{2}C_G\left(gh_0\right)^2 = \frac{1}{2}C_G \Delta\varphi_G{}^2 \tag{2.18}$$

2.4.3 Storing Energy in Pressure Vessels

The derivation for the change of energy of a pressure vessel resulting from the change of volume of liquid stored in it, proceeds along similar lines to what we just did. Let me do it here in the general form. A pressure vessel is described by its elastance or its (hydraulic) capacitance $C_V(P)$ which, in general, is a function of pressure (see Section 1.4.2). If we add fluid to the vessel at pressure P, there is an energy current equal to PI_V accompanying the current of liquid. The integral of this energy flux over time equals the energy communicated to the tank which is equal to the change of energy stored:

$$\Delta E = \int_{t_0}^{t_f} I_E \, dt = \int_{t_0}^{t_f} P\dot{V} \, dt = \int_{t_0}^{t_f} P C_V \dot{P} \, dt$$

or, after a transformation of the integral,

$$\Delta E = \int_{P_0}^{P_f} C_V P \, dP \tag{2.19}$$

If we consider the case of constant capacitance, this results in

$$\Delta E = \frac{1}{2}C_V\left(P_f^2 - P_0^2\right) \tag{2.20}$$

Compare this to Equ.(2.18). We see that it is equivalent to what we obtained for a straight walled open tank in the gravitational field which corresponds to C_G = const.