Prandtl number:

$$Nu = f\left(x^*, Re_L, Pr\right) \tag{14.19}$$

For example, the equations can be integrated for laminar flow over a flat plate:

$$C_{f,x} = 0.664 Re_x^{-1/2} \quad , \quad Nu_x = 0.332 Re_x^{1/2} Pr^{1/3} \tag{14.20}$$

This holds as long as the Prandtl number is larger than 0.6.[3] The subscript x refers to the position in x-direction along the flat plate. The local Nusselt number Nu_x has to be integrated if we wish to obtain the average value up to position x. For this case, it turns out that the average value up to position x is twice the local value at x.

Naturally, other geometries and flow conditions lead to different and often much more complicated problems. Turbulent flow, for example, cannot be treated analytically, at least not without the introduction of additional strong assumptions. Turbulent mixing in the boundary layer leads to greatly increased thermal and momentum diffusivities[4] for which we do not have simple expressions, since they depend upon the state of motion and not just upon fluid properties. In many heat transfer applications of practical interest, experimental determination of the heat transfer coefficient is required. Fortunately, the dimensionless groups help to reduce the complexity of the problem just as they did in the theoretical example. In other words, as in the case of an analytical calculation, we should try to measure the Nusselt number in terms of the Reynolds and the Prandtl numbers to obtain empirical relations analogous to what we have seen in Equ.(14.20).

QUESTIONS

1. How is the entropy transfer coefficient at an interface defined?
2. The equations of balance, Equations (14.4), contain time derivatives. Why are they absent from Equations (14.8) and Equ.(14.9)?
3. In the derivation of Equ.(14.9), four components of the momentum current density tensor are listed. Why are there four components? How can the forms be motivated?
4. What is the importance of the non-dimensional groups introduced to describe boundary layer flows?

14.2 A STUDY OF SOLAR COLLECTORS

Basically, any object which absorbs the light of the Sun is a solar collector. Therefore, we should include in this list the leaves of trees, buildings, soil, the oceans, or photovoltaic cells, just to name a few. You can see the range of phenomena induced by the Sun's rays. The "collectors" may produce substances, they may lead to the flow of electric charge, or they may produce heat (i.e., entropy). Here we shall look only at the latter effect in simple technical devices.

3. See, for example, Incropera and DeWitt (1981), p. 313–318.
4. Incropera and DeWitt (1981), p. 293–296.

A thermal solar collector is a device which absorbs a part of the solar radiation falling upon it, leading to entropy production in the absorber. If we let a fluid flow across the absorber, we may harness some of the entropy which has been created (Fig. 14.2). In the simplest possible geometrical arrangement, we have a flat absorber plate possibly made out of some thin metal sheet. We may then let a fluid (liquid or air) flow through a rectangular duct behind the absorber, with the fluid wetting its entire surface area. Commonly, the collector is insulated at the bottom, and a transparent cover is placed above the absorber plate; both devices reduce the loss of entropy to the environment. The ducts for fluid flow in the collector are often different from (and more complicated than) what we have assumed here, leading to more difficult geometrical arrangements for the transfer of heat between an absorber and fluid. This point will be further discussed below.

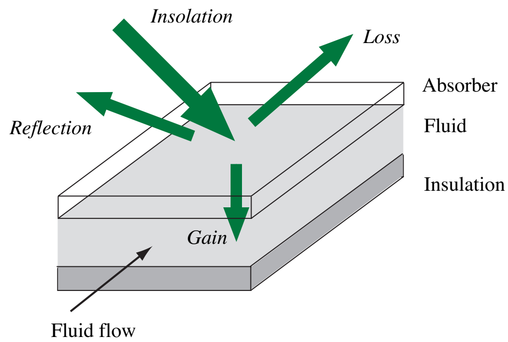

Figure 14.2: Simple flat-plate solar collector consists of a flat absorber plate (possibly including a cover for reducing the losses to the top). Absorption of solar radiation leads to the production of entropy in the plate. A fluid flowing across the bottom of the plate can carry away some of the entropy. The figure shows the simplest possible geometry for absorber and fluid flow. The fat arrows denote the energy fluxes with respect to the absorber.

Basically, a thermal solar collector of the type discussed here is a flow heater (see Chapter 8 for a treatment of uniform models of flow heaters). Unlike the models introduced in Chapter 8, the ones created here will take into account the spatial variation of temperature of the fluid in the duct.

14.2.1 The Balance of Energy for a Solar Collector

Fig. 14.2 shows the fluxes of energy with respect to the absorber. They will be used to express the balance of energy (and of entropy) with respect to the absorber. In a second step, we will perform the balance of energy with respect to the entire collector. First, we have the flux associated with solar radiation falling upon the collector.[5] As you know, unless the surface is a black body, only part of the radiation will be absorbed, while the rest will be reflected back to the environment. The ratio of radiation absorbed to radiation falling upon the surface defines the optical properties of a collector which depend upon the absorber, the cover, and the type of radiation. Usually, for the purpose of an overall balance, their combined effect is described by a factor known as the *transmission-absorption product* $(\tau\alpha)$; see Section 9.4.1 and Chapter 16.

The losses of the collector plate to the environment, on the other hand, are calculated

5. Naturally, radiation falling upon the collector includes the radiation of the atmosphere as a body at or near environmental temperature. This contribution is not included with the solar irradiance $\mathcal{G}$, but rather with the losses of the collector to the environment.

in terms of a heat loss coefficient U_t (top loss coefficient). In the following analysis, we shall assume losses to occur only through the top of the collector. The last energy flux is due to convective transfer at the absorber-fluid interface. To describe its effect, we introduce the heat transfer coefficient from the plate to the fluid (abbreviated by U_{pf}). All three coefficients characterizing a collector, i.e., $(\tau\alpha)$, U_t, and U_{pf}, may depend upon the conditions under which it is operated.

If we collect all the terms, a steady-state balance of energy for the absorber takes the form (Fig. 14.3):

$$(\tau\alpha)\mathcal{G} - U_t\left(T_p - T_a\right) = U_{pf}\left(T_p - T_f\right) \quad \textbf{(14.21)}$$

The indices p and f refer to the absorber plate and the fluid, respectively. As usual, a stands for *ambient*. $\mathcal{G}$ is the total irradiance with respect to the surface of the collector. This equation holds for every point of the surface, with the temperatures of the fluid and the plate changing in the direction of fluid flow (but, for our simple geometry, not in the direction perpendicular to the flow). Naturally, we have a problem concerning the meaning of the temperatures T_f and T_p if we want to apply Equ.(14.21) directly for the entire collector of surface area A. In this case, just think of the temperatures as some appropriate average value for the respective system.

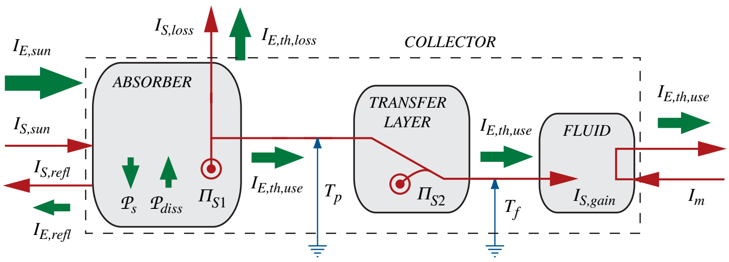

Figure 14.3: Process diagram for the collector showing fluxes of entropy and energy. Almost all the entropy transferred to the fluid and to the environment is produced in the system.

The equation of balance of entropy will be written below for the collector as the system. First, however, let us express the overall balance of energy. In this case, we have to include the convective currents of energy due to fluid flow into and out of the collector, while the flux from the plate to the fluid drops out:

$$c_p I_m\left(T_{f,out} - T_{f,in}\right) = A\left[(\tau\alpha)\mathcal{G} - U_t\left(T_{pm} - T_a\right)\right] \quad \textbf{(14.22)}$$

As just mentioned, in this equation T_{pm} represents the proper average value. Normally, we take both the temperature coefficient of enthalpy of the fluid and the pressure as constants, which permits us to use Equ.(8.60) for the convective current.

Since the temperature of the collector plate is not easily accessible, one replaces the temperature of the absorber in Equ.(14.22) using Equ.(14.21). As a result, the (average) temperature of the fluid appears in the law of balance:

$$I_{E,use} = A\frac{U_{pf}}{U_{pf} + U_t}\left[(\tau\alpha)\mathcal{G} - U_t\left(T_{fm} - T_a\right)\right]$$

$I_{E,use}$ is the useful energy current, i.e., the convective current with respect to the collector. The factor multiplying the term in parenthesis is called the *collector efficiency factor* and it is commonly abbreviated F'. Using this definition leads to the expression for the useful energy current in terms of the fluid temperature:

$$I_{E,use} = AF'\left[(\tau\alpha)\mathcal{G} - U_t\left(T_{fm} - T_a\right)\right]$$
$$F' = \frac{U_{pf}}{U_{pf} + U_t} \qquad \textbf{(14.23)}$$

If you rewrite the efficiency factor in a slightly different form, you can see that it corresponds to the ratio of the thermal resistances between the absorber and the environment on the one hand, and between the fluid and the environment on the other.

Different fluid duct geometries. Usually, the geometrical arrangement of the fluid ducts is different from the simple case assumed so far. This holds especially for collectors using liquids for heat transfer. Therefore, if U_{pf} is still used for the heat transfer coefficient from metal to fluid, the collector efficiency factor F' cannot be computed as done in Equ.(14.23). Take the case of a liquid flowing through thin pipes attached to the absorber. Obviously, heat has to flow through the absorber sheet to the pipes before it can enter the fluid. The efficiency will be reduced both because of this process and because of the fact that the fluid surface may be smaller than that of the flat absorber. The efficiency factor will depend not only upon the heat transfer coefficient, but also upon the distance between the pipes, the thickness and the conductivity of the absorber sheet, and the inner surface area of the pipes carrying the fluid. Therefore, the task of computing the efficiency factor can be quite complicated.[6] However, with a known efficiency factor, a reduced heat transfer coefficient can be calculated and used as in the equations presented above (Example 14.2).

14.2.2 Temperature Distribution in the Direction of Fluid Flow

Equ.(14.23) holds only with the appropriate average of the temperature of the fluid as an overall balance of energy. We may approximate this value by the arithmetic average of the fluid temperatures at the inlet and the outlet of the collector (as shown in Example 14.1). A better expression, however, requires considering the change of temperatures in the direction of fluid flow in the collector. This is achieved by treating the example as a continuous problem in one spatial dimension (Fig. 14.4).

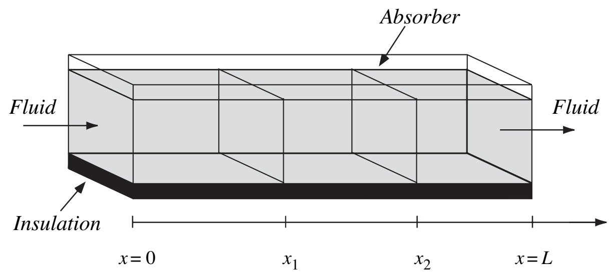

Figure 14.4: Fluid flowing through the collector heats up in the direction of flow. The law of balance of energy for the control volume between x_1 and x_2 leads to an expression for the temperature as a function of position.

6. See for example Duffie and Beckman (1991), Chapter 6.

The law of balance of energy for the control volume of length x_2–x_1 can be used to derive the expression for the temperature of the fluid as a function of position x. The fluid portion in the control volume is heated from above, and it flows into and out of the volume. If we use Equ.(14.23) in the form

$$j_{E,use} = F'\left[(\tau\alpha)\mathcal{G} - U_t\left(T_f(x) - T_a\right)\right]$$

the law of balance for the control volume is

$$c_P I_m\left(T_f(x_2) - T_f(x_1)\right) = F'\int_{x_1}^{x_2} W\left[(\tau\alpha)\mathcal{G} - U_t\left(T_f(x) - T_a\right)\right]dx$$

Here, W is the width of the collector. The left-hand side represents the convective current due to the fluid entering and leaving, while the right-hand side is the integral of the energy current density over the top surface area of the control volume. The equation transforms into the differential equation

$$c_P I_m \frac{dT_f}{dx} = F'W\left[(\tau\alpha)\mathcal{G} - U_t\left(T_f(x) - T_a\right)\right] \tag{14.24}$$

whose solution is

$$T_f(x) = \frac{(\tau\alpha)\mathcal{G}}{U_t} + T_a - \left[\frac{(\tau\alpha)\mathcal{G}}{U_t} + T_a - T_{f,in}\right]\exp\left(-\frac{F'U_t W}{c_P I_m}x\right) \tag{14.25}$$

If we take the value of T_f at the outlet, subtract it at the inlet, and multiply by the product of c_P and I_m, we obtain the useful energy current:

$$I_{E,use} = A\frac{c_P I_m}{AU_t}\left[1 - \exp\left(-\frac{F'U_t A}{c_P I_m}\right)\right]\left[(\tau\alpha)\mathcal{G} - U_t\left(T_{f,in} - T_a\right)\right]$$

Basically, this result represents a transformation of the expression used previously in Equ.(14.23) where the mean fluid temperature is replaced by the temperature of the fluid at the inlet to the collector. As a consequence, the collector efficiency factor F' is replaced by the *heat removal factor* F_R:

$$I_{E,use} = AF_R\left[(\tau\alpha)\mathcal{G} - U_t\left(T_{f,in} - T_a\right)\right] \tag{14.26}$$

where

$$F_R = \frac{c_P I_m}{AU_t}\left[1 - \exp\left(-\frac{F'U_t A}{c_P I_m}\right)\right] \tag{14.27}$$

Equ.(14.26) allows us to calculate the useful energy current of a flat-plate solar collector in terms of environmental parameters, the current of mass through the collector, and the inlet temperature of the fluid. The collector can be characterized by two parameters, $F_R(\tau\alpha)$ and $F_R U_t$; considering that F_R includes the efficiency factor, we may say that there are three values defining the device—$(\tau\alpha)$, U_t, and F'. In practical cases, these parameters are measured (Example 14.1).