The Continuum Physics Paradigm in physics instruction
III. Using the Second Law

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ABSTRACT: An introductory college physics course has been designed, implemented, and taught for several years which combines the continuum physics paradigm with systems thinking and system dynamics tools for modeling and simulation of dynamical processes. In short, it provides an explicit general modeling strategy applicable to all fields of physics and even to fields outside of this science, allowing for student centered (learner directed) learning. The fundamental ideas of continuum physics can be cast in the form of a simple graphical image which is borrowed from the flow of water at the surface of the Earth, and which can easily be translated into system dynamics models of processes. This unified approach to physical processes significantly revises the standard model of physics courses, adds an important methodological dimension not commonly used in physics instruction, and places physics beyond its own borders together with other sciences, engineering, and social studies. It makes use of phenomenological primitives, and it deals with, and proposes a practical solution to, conceptual problems identified in standard courses over the last few decades.

This last paper in a series of three combines the continuum physics view of natural processes with system dynamics modeling to produce a practical version of a theory of the dynamics of heat applicable to introductory physics. It treats dynamical processes in analogy to mechanics by using the law of balance of entropy (the Second Law) in place of Newton's second law. The resulting theory of heat is simple yet strong and general enough to form a natural basis for modern applications in engineering (second law analysis and optimization) and theoretical developments in continuum thermodynamics.

I Introduction

In the first two papers of this series, the foundations of the Continuum Physics Paradigm (CPP) and of system dynamics modeling in physics instruction have been discussed.[1,2] Here the general ideas are applied to a special field—thermodynamics. A theory of the dynamics of heat[3] is outlined and examples of dynamical thermal processes are presented in analogy to mechanics. The presentation will make use of system dynamics modeling throughout.

Classical thermodynamics is a theory of the statics of heat. According to Callen[4], "The single, all-encompassing problem of thermodynamics is the determination of the equilibrium state that eventually results after the removal of internal constraints in a closed, composite system."

Imagine this statement to be made about mechanics, and engineers being allowed to compute only equilibrium states. Naturally, practical thermodynamics is long past this state, but the teaching of thermal physics still gives the impression that equilibrium is all we can hope for. We have to contend with a type of mathematics unknown to any other branch of physics, and with "heat transfer" taught separate from "thermodynamics."[5]

A unified modern presentation of thermodynamics would call for a dynamic theory of heat in analogy to mechanics and other fields of physics. We would formulate and solve initial (and boundary) value problems to obtain the functions of time which describe the evolution of thermal systems.

A modern version of thermodynamics can be based upon the approach afforded by the Continuum Physics Paradigm. The CPP suggests a simple graphical image of the role of entropy in thermal processes, an image based on a comparison to substances in chemistry, charge in electricity, or momentum in mechanics. Entropy is responsible for making a stone warm or for melting a block of ice. It flows into and out of bodies, and it is stored there. It is produced in irreversible processes such as friction, electrical conduction, diffusion, burning of fuels, emission and absorption of radiation, and heat transfer. In summary, entropy is the extensive thermal quantity which satisfies a law of balance (the Second Law). Since laws of balance—augmented by constitutive laws for particular processes—constitute the basis of any theory of dynamics,[1,3] the Second Law is at the core of a dynamical theory of heat.

Here we will present examples of the dynamics of heat which will make uninhibited use of the entropy principle. Thermodynamics is constructed in analogy to introductory mechanics. In mechanics, we create free-body diagrams by identifying all momentum flows (forces) with respect to a body. We know that the sum of all forces determines the rate of change of momentum. By expressing the forces with the help of constitutive laws and solving the differential equation represented by Newton's Second Law, we obtain the velocity and other kinematic quantities describing the motion of the body.

Translating this to thermal physics means that we have to find all entropy flows and the rate of production of entropy to determine the rate of change of the entropy of a body. The balance of entropy plus thermal constitutive laws for the storage, the flow and the production of entropy lead to an initial value problem whose solution is the temperature as a function of time.

Section II will discuss introductory mechanics as a guide to a theory of the dynamics of heat. Section III discusses the properties of entropy and introduces the Second Law and the relation of entropy, temperature, and energy. In the following sections we will discuss special thermal constitutive laws. First, in Section IV, the storage and the transfer of entropy will be dealt with. In Section V, entropy production in heat transfer will be outlined. This topic is important if we want to deal with real life dynamical thermal processes. Finally Section VI will outline applications to heat engines.

By using entropy in this straight-forward manner we hope to provide students and teachers with a rich set of examples of direct use of the Second Law. It would be a shame if in our introductory courses thermal physics did not rise to the level of application and clarity of concept we take for granted in classical mechanics or electricity.

II Introductory mechanics as a guide to a practical theory of the dynamics of heat

Newtonian mechanics is the most extensively treated single subject of introductory physics. Still, the form of the theory is not normally used as an example of how other fields can be structured and taught. Rather, it is suggested that the phenomenon of motion—if transferred to particles in our theories of kinetic theory and statistical physics—explains much of the rest of the physical world, from electricity to heat. In other words, it is assumed that behavior—motion—and not structure—Newton's law combined with constitutive force laws—serves as an adequate description of the world around us.

If we take the structure of mechanics as a guide, we are led to create theories of fluids, electricity, and heat having analogous forms.[1,3] Thermodynamics profits from this approach most dramatically. In the following paragraphs, a slightly generalized version of Newtonian mechanics is presented to demonstrate the structure we will use for the dynamics of heat.

A. An example: Rocket motion

Consider the vertical motion of a rocket at the surface of a planet. The motion of the object is described and explained in terms of the balance of momentum. In a system dynamics model[2] we first introduce a stock to account for the momentum of the system (Fig.1). Then we think of the processes which are the cause of the change of the momentum: gases flowing out of the rocket at the back, air resistance, and the influence of the gravitational field. Each process leads to the exchange of momentum between the system and its environment. We therefore introduce three separate flows representing momentum fluxes and source rates (the flows in Fig.1) in our model.

Since the rocket is an open system, we have to account for the balance of mass as well. The mass of the rocket changes as a consequence of the outflow of gas (Fig.1).

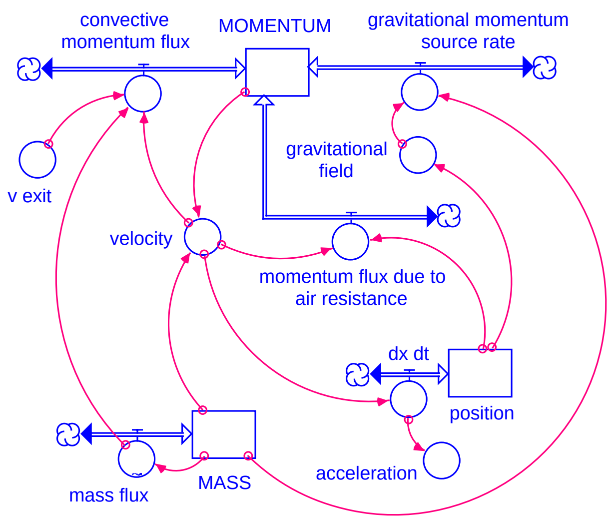

Figure 1: System dynamics model of vertical rocket motion. Note the structures of stocks and flows which represent laws of balance of momentum and of mass. The stock and the flow relating *dx/dt* to the position, on the other hand, denotes a simple integrator. Note the capacitive law relating velocity to momentum and mass. The model was produced and simulated in Stella.[6]

Next we have to determine the fluxes. The current of mass may be prescribed as a function of time whereas the momentum flows have to be specified by special (constitutive) laws for momentum transports. For example, the convective momentum flux depends upon the flow of gas, and upon the speed of the gas with respect to the observer. The gravitational momentum flux, on the other hand, depends upon the instantaneous mass of the system and the gravitational field at the location of the rocket.

We still lack the crucial dynamical relation between momentum and the velocity of the rocket: the velocity equals the ratio of the instantaneous values of momentum and mass of the system. This is called a capacitive law.

Finally, we introduce kinematic relations: position and acceleration of the rocket are calculated from the velocity of the object (Fig.1). Position and acceleration are obtained by integration and differentiation of the velocity, respectively.

The finished model can be simulated to obtain the motion of the rocket. Parameters may be changed, and the model may be extended to account for other circumstances. If desired, energy quantities can be included in the model.

B. Dynamics: Laws of balance and constitutive laws

The system dynamics diagram of the model of rocket motion (Fig.1) reveals the structure of the theory of mechanics used to solve the problem. A theory of dynamics in mechanics rests upon the law of balance of momentum and associated constitutive laws for momentum transports, and on the capacitive law relating velocity to momentum. The general law of balance of momentum takes the form

$$\dot{p} = \sum_{i=1}^{n} I_{p,i} \quad \textbf{(1)}$$

In our example, we are dealing with all three modes of transport commonly found in nature: conductive (due to air resistance), convective (as a result of gas flow), and radiative (due to the interaction of the body and the gravitational field). The first two transports lead to momentum fluxes with respect to the system, whereas the third is described in terms of a source rate. In our model, these quantities are given by

$$\begin{aligned} I_{p,res} &= -const \cdot density(position) \cdot velocity^2 \\ I_{p,conv} &= (v_{exit} + velocity) \cdot mass\,flux \\ I_{p,grav} &= -gravitational\,field(position) \cdot mass \end{aligned} \quad \textbf{(2)}$$

The first and the third of the rates of transport are normally called forces (the forces of air resistance and of gravity).

In a given model, there are particular laws not necessarily encountered in models of processes of other fields of physics. In mechanics, for example, the geometric relations of kinematics play a role which are not found in simple thermal or electric processes.

In summary we can say that models of mechanical dynamical processes are set up as a combination of the law of balance of momentum and associated constitutive laws.[1] Together these equations constitute the evolution equations of the mechanical system. Simulation of the equations yields the behavior of the system over time.

C. The energy principle

Energy accompanies all physical processes. It plays a unique role: it relates different types of processes, i.e., it provides the "glue" in chains of processes.[1] Take for example the chain leading from a waterfall, through a turbine and generator, to the electric heating of water. Each process in the chain is governed by a substancelike quantity such as the volume of water, angular momentum, charge, or entropy, and the associated potentials. In this chain, energy determines how one process is related to the next. For example, in an ideal turbine, the hydraulic power (i.e., the rate at which energy is released by the fall of water from a point of high to a point of low pressure) is equal to the power of the rotational process.

In a mechanical process, momentum takes the role of the "energy carrier." Thus the rate at which energy is released is determined by the momentum current flowing through a velocity difference. Take as an example two bodies rubbing against each other. Momentum flows from the faster to the slower body, releasing energy at the rate

$$\mathcal{P} = -\Delta v \left| I_p \right| \quad (3)$$

Because of its role, the energy principle generally yields valuable additional information for constitutive theories. In our example of rocket motion, we may use knowledge of the amount of energy released by the burning of fuel to obtain information on the conditions in the engine, and, consequently, on the flow speed of the gases. Here, energy relates the chemical process to the mechanical one.

D. The $F = ma$ of thermal physics

The teaching of introductory mechanics is quite sophisticated in many courses. Teachers make use of the structure of the theory to clearly explicate how problems must be solved.[7] We choose systems, create free body diagrams and identify forces, determine forces by forces laws, add them up and set them equal to mass times acceleration. With the help of kinematic relations we finally figure out the motion of the body.

If mechanics can be a guide for thermodynamics, a theory of the dynamics of heat must rest on the proper law of balance for thermal processes, i.e. the equivalent of Newton's Second Law. As Newton's law is expressed in terms of a "quantity of motion," we must find a "quantity of heat," i.e., the thermal quantity analogous to momentum, to build thermal physics upon.

If we can find "Newton's Law of thermal processes," we can use it by expressing the fluxes, source rates, and production rates it may contain with the help of constitutive laws. As in mechanics, these equations are the evolution equations of the thermal system. Simulation of the equations yields the behavior of the system over time.

III Entropy and the Second Law

Entropy turns out to be the quantity analogous to momentum, and the law of balance of entropy is "Newton's Law of thermal processes."

In this section, the properties of entropy will be discussed, the balance of entropy will be formulated, and the relation between entropy, temperature, and energy will be introduced.

We can make use of strong phenomenological primitives to motivate entropy as a "quantity of heat" or the "thermal charge"[8] to beginners from the start.[9] Moreover, if the energy principle has been discussed in a couple of introductory fields such as hydraulics and electricity, students will find it easy to relate entropy to temperature and to energy. The latter relation yields important information which allows us to calculate rates of production of entropy, a crucial task if we want to make use of the Second Law.

A. The properties of entropy

We introduce a physical quantity which we hold responsible for making a stone warm, or for melting a piece of ice. It goes into a body of air to let it get warmer or to increase its volume. It is stored in bodies, it can flow, and it is produced in irreversible processes. In other words, it is the extensive thermal quantity that satisfies a law of balance. It is in contrast to temperature which measures the intensity of heat. Moreover, as a measure of a quantity of heat, it cannot be energy. Energy makes a body heavier and more inert, not warmer.

We call this quantity *entropy*. In everyday language we would call this quantity *heat*.

Recognizing that entropy can be introduced in this graphical and straight-forward manner—in analogy to the role of fluids in hydraulics, electric charge in electricity, or momentum in processes involving motion—we can use it right from the start. The law of balance of entropy takes a role equivalent to that of Newton's Second Law in mechanics. It lets us state initial value problems which are used to compute dynamical thermal processes.

B. The balance of entropy

Entropy can be stored in bodies, it can flow into and out of bodies, and it can be produced in irreversible processes. Therefore, we expect it to satisfy a law of balance of the form

$$\dot{S} = \sum_{i=1}^{n} I_{S,i} + \Pi_S \qquad \textbf{(4)}$$

It has the same basic structure as the law of balance of momentum, which is called Newton's Second Law. The law of balance of entropy is called the *Second Law of Thermodynamics*. It is the law we need to start a theory of the dynamics of heat (Fig.2).

The fluxes in the laws of balance may be the result of conduction or convection. Source rates have been neglected. Note that irreversible processes call for the rate of production of entropy which may not be negative:

$$\Pi_S \geq 0 \qquad \textbf{(5)}$$

If we can express the currents of entropy and the rate of production of entropy for a process, we may calculate the behavior of the system under investigation.

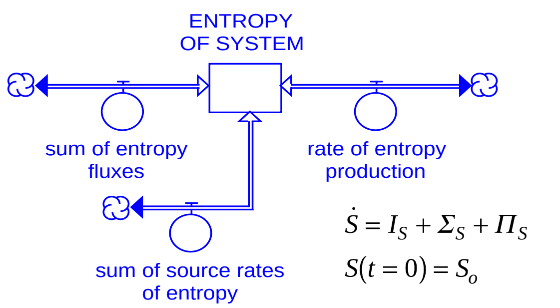

Figure 2: The law of balance of entropy, i.e., the Second Law, expressed as a system dynamics diagram. The stock represents the instantaneous value of the entropy stored in a system whereas the pipelines stand for entropy currents or entropy production rates.

C. Thermal constitutive laws

By themselves, equations of balance are not of much use. Looking at the graphical representation of the law of balance of entropy in Fig.2 we see what is missing: we need special laws to determine the flows and rates of production. These laws are called constitutive relations. They distinguish between different circumstances whereas the law of balance always retains the same form. Constitutive laws are found for relations between entropy storage and temperature (in the case of a stone), between entropy, temperature, and volume (in the case of air), for entropy transfer, and for entropy production. (See Sections IV and V.)

D. Thermal processes: Entropy, temperature, and energy

Energy provides important information for thermal constitutive theories through its relation with entropy and temperature. The first who recognized this important fact was Sadi Carnot. He proposed to develop a complete theory of heat by analyzing the motive power of heat engines.[10] The relation takes three forms, one for the fall of entropy from points of high to points of low temperature, one for the production of entropy, and one for the flow of entropy in heating or cooling.

Thermal power. In a thermal process with a current of entropy flowing from a point of temperature T_1 to a point where the temperature takes the value T_2, energy is released at the rate

$$\mathcal{P}_{th} = -\left(T_2 - T_1\right)\left|I_S\right| \qquad \textbf{(6)}$$

(Fig.3). This quantity is called thermal power. We accept this relation as one of the fundamental rules of thermal physics. We may motivate it on the basis of analogies to other fields of physics,[3] just as Carnot had done in his image of the power of heat. For more advanced students, the relation can be derived on the basis of additional assumptions such as those involving the properties of the ideal gas.[11]

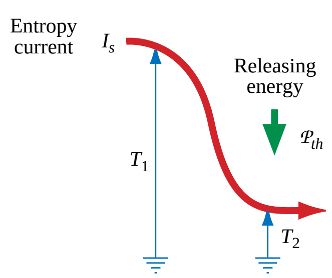

Figure 3: The relation between entropy, temperature, and energy can be motivated on the basis of the image of heat created by Carnot. He compared caloric to water falling through a certain height, thereby releasing energy at a well-defined rate. Caloric is taken to represent entropy.

Entropy production and dissipation. Entropy appears out of nothing, but we have to work to produce it. The energy released to be bound in the production of entropy is said to be dissipated (Fig.4). The rate of dissipation $\mathcal{D}$, and the rate of production of entropy are related by[12]

$$\mathcal{D} = T\Pi_S \tag{7}$$

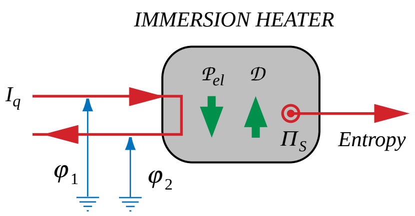

Figure 4: System diagram of an immersion heater showing dissipation and production of entropy.

The conductive transport of entropy and energy in heating and cooling. In heating or cooling, the flux of entropy is associated with a flux of energy calculated according to[13]

$$I_W = TI_S \tag{8}$$

(Fig.5). In contrast to the rate at which energy is released (which is called power) the quantity expressed in Eq.8 is called an *energy current*.

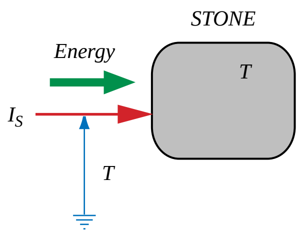

Figure 5: Entropy flux and energy flux in heating of a body at temperature *T*.

IV Storage and transport of entropy

Let us now consider simple constitutive theories of heating of uniform bodies (stones, water, air), and of entropy transfer into and out of these bodies.

A. Uniform heating at constant volume

Thermal dynamical theories need thermal constitutive laws. The model of heating of uniform bodies shows how to introduce the quantity called entropy capacity in the case of heating and cooling at constant volume:[14]

$$\begin{aligned} \dot{S} &= K(T)\dot{T} \\ \Delta S &= K\Delta T \quad \text{if } K = const \\ K &= C/T \end{aligned} \tag{9}$$

The entropy capacity K is related to the temperature coefficient of energy C by the temperature of the body.[15] Specific temperature coefficients of energy are listed in tables.

Uniform heating proceeds reversibly.[14] If a body is to remain uniform during heating or cooling, it must be a perfect conductor. In a perfect conductor, there is no production of entropy. Therefore,

$$\Pi_S = 0 \tag{10}$$

and

$$\dot{S} = I_S \tag{11}$$

B. Thermodynamics of ideal fluids: The ideal gas

A simple fluid such as the ideal gas reacts to heating by changing either the temperature or the volume. We introduce the entropy capacity at constant volume K_V and the latent entropy with respect to volume Λ_V by[16]

$$\dot{S} = \Lambda_V \dot{V} + K_V \dot{T} \tag{12}$$

Again, the uniform heating of an ideal fluid is reversible. Neither the flow of heat—which is taken to be perfect—nor changes of volume produce entropy.

Consideration of a Carnot cycle of the ideal gas demonstrates that[17]

$$\Lambda_V = \frac{nR}{V} \tag{13}$$

If we express the heating of the ideal gas in terms of pressure and temperature, i.e., if we write

$$\dot{S} = \Lambda_p \dot{P} + K_p \dot{T} \tag{14}$$

we also introduce the latent entropy with respect to pressure, and the entropy capacity at constant pressure. Transformation of variables with the help of the equation of state of the ideal gas

$$PV = nRT \tag{15}$$

leads to an expression for the entropy capacity at constant volume in terms of the ratio $\gamma = K_P/K_V$ of the entropy capacities:

$$K_V = \frac{nR}{\gamma - 1}\frac{1}{T} \quad (16)$$

The ratio of the entropy capacities can be measured in some experiments. The model of the Rüchardt experiment presented below in Section IV.E demonstrates how this can be done. Therefore, theory (the consideration of the Carnot cycle), and measurement of one quantity yield the two constitutive quantities needed to compute processes involving the ideal gas.

The quantities $C_V = TK_V$ and $C_p = TK_p$ are called the temperature coefficient of energy and the temperature coefficient of enthalpy, respectively.[15] They are listed in tables. Note that the ratio of the entropy capacities is equal to the ratio of the temperature coefficients.

C. Overall entropy transfer

We do not only need constitutive laws relating the entropy of a system to other system variables. We also have to be able to explicitly compute transfer rates of entropy with respect to a given body.

The simplest expression for a current of entropy in an entropy transfer process is the one involving the temperature difference $\Delta T = T - T_a$ which drives the process, and an entropy conductance G_S:

$$I_S = G_S \Delta T \quad (17)$$

The conductance generally is proportional to the surface area through which the entropy is transferred. The factor multiplying the area A is called the overall entropy transfer coefficient U_S:

$$I_S = AU_S \Delta T \quad (18)$$

Note that—as a result of entropy production in entropy transfer (Section V)—we have to be precise about where the entropy current is measured. The entropy transfer coefficient is taken at the same location, and it may be a function of the temperature at that location. From the relation between entropy and energy current in heating and cooling (Eq.8) we find that

$$\begin{aligned} I_W &= AU\Delta T \\ U &= TU_S \end{aligned} \quad (19)$$

The energy current associated with the flow of entropy can be expressed in terms of the energy transfer coefficient U. This constitutive quantity is related to the entropy transfer coefficient by the temperature of the surface through which heat flows. It may be found listed in tables for some common circumstances.

D. Conduction, convection, and radiation

Special constitutive theories of the different modes of entropy transfer are needed if we wish to quantify thermal dynamical processes.[18] Heat transfer in general is irreversible and is associated with the production of entropy (Section V).

The conductive current of entropy through a plane slab of matter is proportional to the temperature difference across the slab, the surface area, and to the entropy conductance of the material. It varies inversely with the thickness:

$$I_S = Ak_S \frac{\Delta T}{\Delta x} \quad (20)$$

Here, $k_S = k/T$ is the entropy conductivity, and k is the well known thermal conductivity. The entropy current at a solid-fluid interface is expressed by

$$I_S = Ah_S \Delta T \quad (21)$$

$h_S = h/T$ is called the convective entropy transfer coefficient.

The entropy current carried by a current of fluid of temperature T_f having a constant temperature coefficient of enthalpy is

$$I_{S,conv} = c_p I_m \ln\left(T_f / T_{ref}\right) \quad (22)$$

where T_{ref} is an arbitrary reference temperature. c_p is the specific temperature coefficient of enthalpy. The energy current carried by such a fluid is

$$I_{W,conv} = c_p I_m \left(T_f - T_{ref}\right) \quad (23)$$

Note that for convective currents, Eq.8 does not hold.

Hot bodies emit electromagnetic radiation which carries with it some entropy. If the radiation is that of a black body, the surface emits an entropy current

$$I_{S,rad} = \frac{4}{3} A\sigma T^3 \quad (24)$$

The energy current associated with the radiation is

$$I_{W,rad} = A\sigma T^4 \quad (25)$$

E. Models of uniform dynamical processes

The constitutive laws presented in this chapter allow for the creation of models of dynamical thermal processes of single uniform systems where the production of entropy due to transfer, mixing, absorption, and other phenomena takes place outside the system.

We will discuss two simple phenomena, one involving the heating and cooling of water, the other adiabatic processes of the ideal gas.

Consider a body of water heated in a glass by an immersion heater, insulated at the bottom and at the top (to prevent evaporation). The water is mixed to ensure uniform conditions. In Fig.6, the system dynamics diagram of a model of the process is presented. We begin with the law of balance of entropy, i.e., with a stock and two flows. The latter represent the entropy source rate due to heating by the immersion heater and the entropy flux due to heat loss, respectively. The capacitive law is expressed in terms of the rate of change of entropy and rate of change of temperature according to Eq.9. The sum of the entropy fluxes equals the rate of change of entropy. Dividing this quantity by the entropy capacity yields the rate of change of temperature which is integrated (the stock labeled *Temperature* in Fig.6). The entropy capacity is obtained by dividing the temperature coefficient of energy by the temperature. Having calculated the temperature, we can now determine the source rate of entropy and the entropy flux due to heat loss from Eqs.7, 17, and 19.

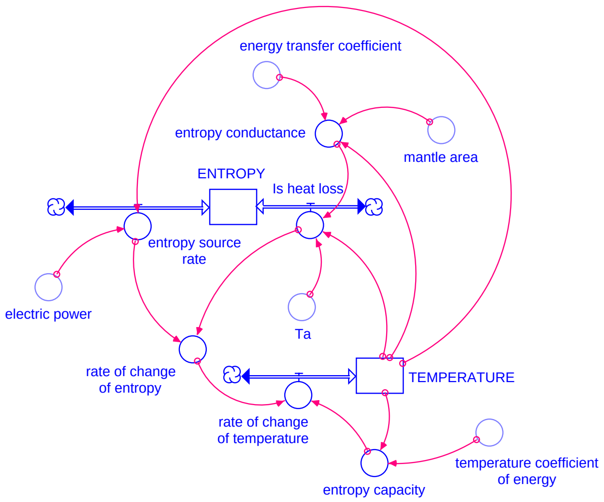

Figure 6: System dynamics diagram of a model of heating of water modeled as a uniform body. The entropy changes due to absorption from an immersion heater, and loss through the mantle of the container. The capacitive law determines the rate of change of the temperature of the body.

The second example is Rüchardt's experiment. A steel ball is dropped into a vertical glass pipe fitted at the top of a large sealed bottle containing air. In this experiment we can measure the ratio of the entropy capacities by observing the steel ball bouncing up and down on a cushion of air in a glass pipe. The model presented in Fig.7 combines the motion of the ball with the dynamics of the air in the bottle modeled as a uniform body.

The motion of the steel ball follows the procedure outlined in Section II. We formulate the law of balance of momentum for the ball. The momentum changes as the result of four interactions: gravity, the push of the air above, the push of the air below, and friction. There are four pipelines connected to the stock representing momentum in Fig.7.

The most interesting of the four mechanical interactions is due to the pressure of the air below the steel ball. This pressure is changing since the volume and the temperature of the air are changing. The relation between pressure, volume, and temperature is determined by the thermal processes undergone by the body of air. These are modeled as follows. We start with the laws of balance of entropy and volume. The entropy can change as the result of entropy flow to or from the surroundings (*entropy flux* in Fig.7).

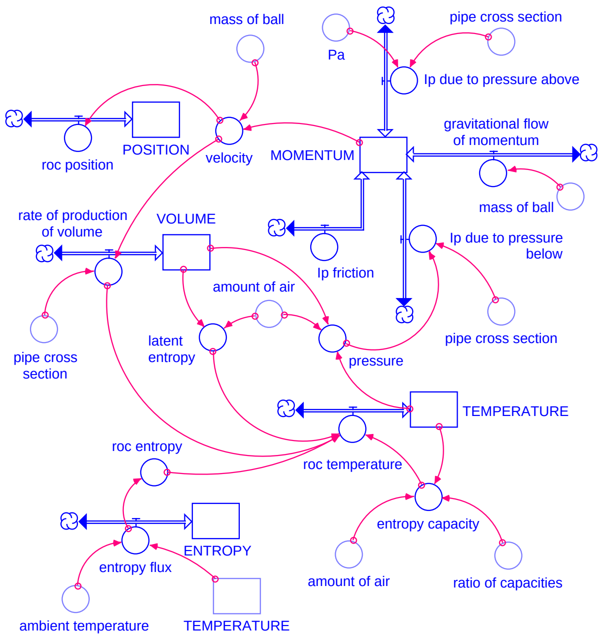

Figure 7: System dynamics diagram for the model of the motion of a steel ball coupled to oscillations of air in a bottle (Rüchardt's experiment). Experiment and model let us determine the ratio of the entropy capacities of air.

The volume changes due to production and destruction resulting from the ball moving up and down (*rate of production of volume* which is proportional to the velocity of the ball). Now we still need to calculate the temperature from the volume and the entropy. This can be done by making use of the fundamental constitutive law, Eq.12, for the ideal gas involving the constitutive quantities of the gas—namely the entropy capacity and the latent entropy. It lets us determine the rate of change of temperature (*roc temperature* in Fig.7). The temperature is obtained by integration. The pressure is then determined by the equation of state, Eq.15.

The model lets us calculate adiabatic oscillations of the gas if we set the entropy flux between gas and surroundings to zero. Otherwise, we see that heat transfer leads to damping of the oscillation, even if mechanical friction has been set to zero. Comparison of measurements and simulation results lets us determine the ratio of the entropy capacities of air.

V Entropy production in heat transfer

Entropy production due to friction, conduction of electricity, and fires, is quite obvious. It is less obvious, however, that entropy is produced also in heat transfer, emission and absorption of radiation, and mixing of fluids.

A. Irreversibility in entropy transfer

The energy dissipated in heat transfer is the energy released due to the fall of entropy from high temperature T_1 to low temperature T_2. The entropy produced appears at the lower temperature T_2. Therefore

$$\Pi_S = \frac{1}{T_2}(T_1 - T_2)|I_{S1}| = \frac{1}{T_2} A U_S (T_1 - T_2)^2 \qquad \textbf{(26)}$$

B. Entropy production in the absorption and emission of radiation

Assume that black body radiation of temperature T_r is absorbed by a body at temperature T. The rate at which entropy is absorbed is denoted by Σ_s. The rate of production of entropy is given by

$$\Pi_S = \left(\frac{3}{4}\frac{T_r}{T} - 1\right)\Sigma_S \qquad \textbf{(27)}$$

In the case of solar radiation at the surface of the Earth we can neglect the second term. In general, however, the expression for the rate of entropy production could turn out to be negative. This means that absorption of radiation must always be accompanied by emission. The rate of entropy production as a result of emission of black body radiation of the body at temperature T is

$$\Pi_S = -\frac{1}{3}\Sigma_{S,e} \qquad \textbf{(28)}$$

where $\Sigma_{S,e}$ is the rate at which entropy is emitted by the body.[19]

C. Entropy production in the mixing of fluids

Let a fluid stream having a mass flux I_m and a temperature T_f enter a system with a fluid having a temperature T. The rate of entropy production by mixing of the incompressible fluids is

$$\Pi_S = c_p I_m \left(\frac{1}{T}\left(T_f - T\right) - \ln\left(\frac{T_f}{T}\right)\right) \qquad \textbf{(29)}$$

The expressions for entropy production derived in this section are the result of the application of the laws of balance of entropy and energy to the situations discussed.

D. Examples of dynamical thermal processes

Here we present two examples. First we treat the case of hot water cooling in a thick walled container which cools to the environment. Second, we will consider the model of the surface of the Earth protected by a layer of atmosphere. In both examples, the bodies will be treated as spatially uniform. These examples show the application of entropy production in entropy transfer and in radiation. See Section V.C of CPP II for an example involving mixing of fluids in a thermal solar collector.

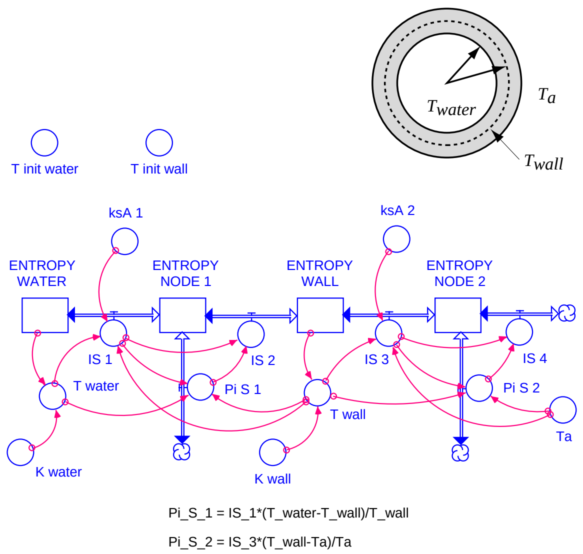

Figure 8: System dynamics diagram of model of cooling of water in a container. Water and container wall are modeled as uniform bodies at temperatures T_{water} and T_{wall}, respectively. Entropy production is associated with transfer layers between water and wall, and wall and environment. The transfer layers are modeled as nodes which do not store entropy.

In the first example, hot water is poured into a cold cylindrical container, and a lid is placed on top. The water is always well-mixed. Even though the container wall will not be uniform, we are going to model it as a thermally homogeneous body at a temperature which is associated with the middle radius (Fig.8). Entropy transfer is due to a layer which is modeled as a node, i.e. a body which does not store entropy. The first layer includes the convective interface between water and wall, and the inner half of the wall. The second transfer layer is made up of the outer half of the wall and the convective interface to the environment. Entropy production is associated with the transfer layers, not with the water or the wall. The equations for entropy production have been derived according to Eq.26. Note that in the model shown in Fig.8, the entropy capacities of water and wall, and the entropy conductances are treated as constant which is approximately valid.

Now consider the second example. Surface and atmosphere are treated as uniform black bodies, and they are given arbitrary initial temperatures. How do the temperatures change as the result of the (average) insolation?

This is an application of the Second Law to the well known example of calculating average surface temperatures in a simple model. We will obtain not only the temperatures of the bodies, but also the magnitudes of the contributions to irreversibility from the different processes. Making it a dynamical model adds a dimension not commonly treated in introductory texts.[20] We can even treat the influence of the daily variation of radiation in a simplified manner.

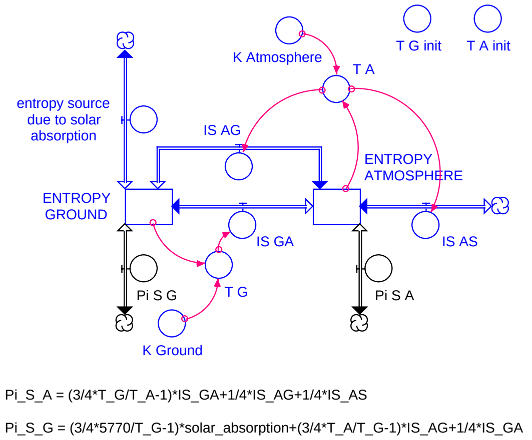

Figure 9: System dynamics diagram of a strongly simplified model which yields average temperatures of the surface of the Earth and the atmosphere. Both systems are treated as uniform black bodies having constant entropy capacities (Eq.9). The rates of production of entropy occurring in the bodies are presented as equations. Note that not all necessary connections are shown in the diagram.

Fig.9 shows the system dynamics model of this example. We express the laws of balance of entropy for the ground and the atmosphere. There are three entropy source rates for the ground—namely those due to solar radiation, emission, and absorption of radiation from the atmosphere. Moreover, there is entropy production in the ground as the result of all three processes (first equation in Fig.9).

The atmosphere also undergoes three interactions. Entropy is absorbed from the surface of the Earth, and it is emitted to the ground and to outer space. The entropy production is again due to all three processes (second equation in Fig.9). In the steady state, we obtain the temperatures known from the simple model based on the balance of energy.[20]

VI Thermal engines

Heat engines, refrigerators, and heat pumps are normally operated at steady-state, and we will treat them here as such. Including irreversibility—mostly due to heat transfer—leads to simple yet useful models.

A. Ideal thermal engines

A heat engine works as follows.[10,13] Entropy flows from the furnace through the engine to the cooler. In steady-state, the energy transferred in the mechanical process equals the thermal power of the fall of entropy if the engine operates reversibly:

$$I_{W,mech} = (T_2 - T_1)|I_S| \qquad \textbf{(30)}$$

Here, T_1 and T_2 are the upper and lower operating temperature, respectively. From this and Eq.8 we can show that the thermal efficiency, i.e., the first law efficiency, is

$$\eta_1 = \frac{T_1 - T_2}{T_1} \qquad \textbf{(31)}$$

Ideal heat pumps and refrigerators simply work in the reverse. We drive the engine, and the energy released in the driving process is used to pump entropy from a lower temperature to a higher one. The thermal power necessary for pumping is calculated as in Eq.30.

B. Irreversibility in thermal engines

Irreversibility in heat engines is the result mainly of entropy transfer into and out of the engines. The working fluid in the engine operates almost reversibly. This leads to the model of endoreversible engines proposed by Curzon and Ahlborn.[21] We can think of the engine to consist of an ideal Carnot engine having two heat exchangers, one at the hot end, one at the cool end.[22] Entropy production is due to entropy transfer from the furnace to the working fluid, and from the working fluid to the cooler. If T_1 and T_2 are the temperature of the furnace and of the cooler, respectively, the power at minimal entropy production is

$$\eta_1 = 1 - \sqrt{\frac{T_2}{T_1}} \quad (32)$$

This example and various practical applications have taught engineers the use of minimization of entropy production as the tool for optimization of thermal systems and processes. To minimize irreversibility, i.e., entropy production, often—but not always—means to maximize the power of a process.

C. The Production of Winds

Here is an example of a steady-state analysis involving minimization of irreversibility. The production of winds may be viewed as the output of a heat engine operating on the surface of the Earth, powered by solar radiation.[23] If the principle of minimal entropy production is applied, an unknown parameter such as the surface temperature of the Earth can be obtained.

Consider the surface of the Earth in steady-state. Despite the rotation of the planet, the daily changes of the surface temperature are small. Therefore assume the surface to be illuminated constantly at a quarter of the solar constant of 1370 W/m^2 of which 70% is absorbed. (The factor 0.25 is the ratio of the cross section of the planet—illuminated at 1370 W/m^2—to the surface are of the sphere). Imagine the atmosphere undergoing constant convective motion which we are going to model as the effect of many endoreversible heat engines. Having many such engines ensures continuous, steady-state operation in the model as well.[24]

What is happening is this (Fig.10). The surface of the Earth absorbs a fraction of the solar radiation. Entropy is produced and part of it is transferred into the atmosphere at low level. The rest is radiated directly toward outer space. Again, entropy is produced. The air moves up cooling adiabatically. At high altitudes, the entropy of the air is radiated to outer space, again accompanied by entropy production. To complete the analysis, we have to add the entropy produced in outer space which we model as a uniform body at 3 K.

We will create a model of an endoreversible engine and search for the condition of minimal total entropy production. There are three laws of balance of entropy of interest in the system in Fig.10:

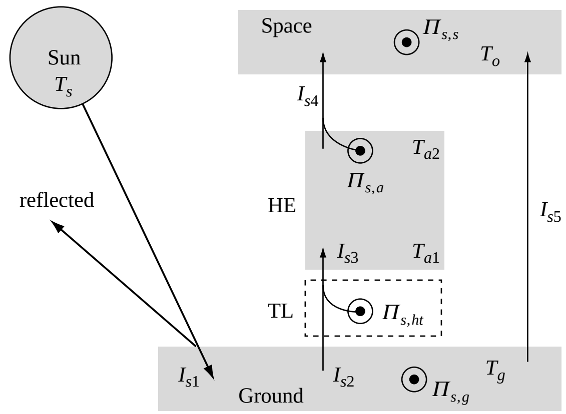

Figure 10: Sun, ground, air, and outer space interact to create a heat engine. HE and TL stand for heat engine and transfer layer, respectively. Note that entropy is produced as the result of different processes taking place in this system.

$$\begin{aligned} I_{S1} + \Pi_{S,g} &= I_{S2} + I_{S5} \\ I_{S2} + \Pi_{S,ht} &= I_{S3} \\ I_{S3} + \Pi_{S,a} &= I_{S4} \end{aligned} \quad (33)$$

The first is for the ground, the second for the heat transfer layer between ground and air, and the third describes the air. All we need now are the constitutive laws for the fluxes of entropy:

$$\begin{aligned} I_{S1} &= 0.25(1 - albedo)\left(\frac{R_s}{d}\right)^2 \frac{4}{3}\sigma T_s^3 \\ I_{S2} &= \frac{h}{T_g}\left(T_g - T_{a1}\right) \\ I_{S4} &= \frac{4}{3}\sigma T_{a2}^3 \\ I_{S5} &= \frac{4}{3}\sigma T_g^3 \end{aligned} \quad (34)$$

and the entropy production rates:

$$\begin{aligned} \Pi_{S,g} &= \left(\frac{3}{4}\frac{T_s}{T_g} - 1\right) I_{S1} + \frac{1}{4} I_{S5} \\ \Pi_{S,ht} &= \frac{1}{T_{a1}}\left(T_g - T_{a1}\right) I_{S2} \\ \Pi_{S,a} &= \frac{1}{4} I_{S4} \\ \Pi_{S,s} &= \left(\frac{3}{4}\frac{T_{a2}}{T_o} - 1\right) I_{S4} + \left(\frac{3}{4}\frac{T_g}{T_o} - 1\right) I_{S5} \end{aligned} \quad (35)$$

R_s and d are the radius of the sun and the distance between Earth and Sun, respectively. T_s denotes the temperature of the Sun. The albedo is the fraction of solar radiation reflected by our planet. T_{a1} and T_{a2} denote the temperatures between which the heat engine producing the winds is operating. They correspond to the temperature of the air close to the ground and at high altitudes, respectively. Reradiation by outer space has been neglected.

Note that the bodies have been treated as black body radiators. Numerical solution of the equations yields a minimum of the total rate of entropy production at T_g = 238 K. This should be compared to an average surface temperature of 288 K.

Finally, the power of the heat engine is computed from the entropy flux through the engine and the upper and the lower temperatures:

$$\mathcal{P}_{HE} = \left(T_{a1} - T_{a2}\right) I_{S3} \tag{36}$$

The predicted power of the wind engine is 18 W/m^2. The real average power of the winds is around 7 W/m^2.

We can combine this model with the one presented in Section V, Fig.9 to include the greenhouse effect of the atmosphere. This changes the situation drastically. Treating the absorbing and reemitting atmosphere as a single layer to produce the simplest model gives results of 283 K and 9 W/m^2 for the ground temperature and the power of the winds, respectively.[25] Remember that these values hold for minimum entropy production. Dividing the atmosphere into a number of different layers, and including the effect of evaporation, can produce a rather useful model which includes not only atmospheric temperatures but also an estimate of the power of the winds. Results come surprisingly close to the actual numbers.

VII Conclusion

Building a theory of the dynamics of heat which can be taught in analogy to the structure of introductory mechanics is not difficult. We can make use of the law of balance of entropy and appropriate constitutive laws for the fluxes and production rates of entropy. In this form, thermodynamics grows up to a level expected from mechanics. We can formulate initial value problems—evolution equations—and solve examples of real dynamical processes in thermal systems.

Making use of the Second Law may appear a bit more complicated than employing Newton's second law because of the appearance of a production term for entropy. On the other hand, simple models of uniform thermal processes do not face the same geometrical difficulties encountered in mechanical applications. Overall, we can expect a theory of the dynamics of heat to be applicable to introductory physics teaching.

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4 H.B.Callen: *Thermodynamics and an Introduction to Thermostatistics*, 2nd ed. (Wiley and Sons, New York, 1985), p.26.

5 In their introductory chapters, books on engineering thermodynamics explain why this subject does not have anything to do with heat transfer, and heat transfer books explain why the subject is not thermodynamics.

6 See Reference 2 for details on system dynamics modeling, diagrams, and tools.

7 A new approach to the teaching of mechanics based upon our knowledge of the modeling structure of physics has been presented by Hestenes and co-workers. D.Hestenes, "Toward a modeling theory of physics instruction," Am. J.Phys. **55**(5),440-454 (1987). D.Hestenes, "Modeling games in the Newtonian World," Am.J.Phys. **60**(8), 732-748 (1992). I.A. Halloun and D. Hestenes, "Modeling instruction in mechanics" Am.J.Phys. **55**(5),455-462 (1987).

8 McGraw-Hill Encyclopedia of Science and Technology.

9 Reference 3; Chapter 1, Section 1.1, p.51-66.

10 S.Carnot: *The Motive Power of Heat*. Carnot proposed the image of a fall of heat from high to low temperature, thereby releasing energy—just as water would. Naturally, he did not use words such as entropy and energy. However, his image can be described today using these words.

11 Reference 3; Epilogue E.1.1, p.592-601.

12 Reference 3; Chapter 1, Section 1.7, p.120-124.

13 Reference 3; Chapter 1, Section 1.4, p.82-100.

14 Reference 3; Chapter 2, Sections 2.1 and 2.2, p.153-170.

15 The temperature coefficient of energy is normally called "heat capacity at constant volume." Since "heat" in the sense of energy does not reside in bodies, the name "heat capacity" is misleading. Analogously, C_P is here called the temperature coefficient of enthalpy.

16 Reference 3; Chapter 2, Sections 2.3-2.5, p.170-206.

17 This is an interesting example of the use of the entropy and energy principles to derive a constitutive quantity which then does not have to be measured in the laboratory.

18 Reference 3; Chapter 3.

19 $\Sigma_{S,e}$ is the rate at which the material body emits entropy. The entropy flux carried away by the radiation field is equal to the sum of this emission rate and the rate of production of entropy calculated in Eq.28.

[20] D. Halliday, R. Resnick, J. Walker: *Fundamentals of Physics*, Fourth Edition, John Wiley & Sons, New York, 1993, Essay E7.

[21] F.L.Curzon, B. Ahlborn, “Efficiency of a carnot engine at maximum power output,” Am.J.Phys. **43**,22-24 (1975).

[22] Reference 3; Chapter 1, Section 1.7, p.131-144.

[23] J.M.Gordon, Y.Zarmi, “Wind energy as a solar-driven heat engine: A thermodynamic approach,” Am.J.Phys. **57**,995-998, 1989.

[24] This condition is different from the one used by the authors of Reference 23. They consider a heat engine which receives energy and entropy for half of the cycle time, using the other half to reject the entropy. Therefore, the energy flux into this engine is twice the average absorbed solar flux of 0.25(1-*albedo*)·*SolarConstant*. Having a larger value of the heating—even if it is for shorter duration—leads to a rather large temperature of the surface, and a large value of the power of the winds (277 K and 17 W, respectively). Such a stop-and-go operation of the heat engine seems unrealistic. A model assuming continuous operation leads to a smaller value of the surface temperature. Indeed, it is smaller than the value of 255 K obtained from the simplest models not including an atmosphere. Allowing for part of the entropy of the ground to be carried away by the air and radiated to space at higher altitudes necessitates a value below 255 K for the surface.

[25] In the enhanced model, we include a body representing the absorbing and reemitting atmosphere in Fig.10. This body is distinct from the air which takes part in convective motion of the heat engine. The atmospheric layer is taken to have an infrared absorptivity of 0.87. There is a new free parameter, the infrared emissivity of the upper layer of air taking part in the wind engine. To obtain the values reported here, a value of 0.25 was chosen for the emissivity. This may be justified on the basis of assuming that it is a thin layer of the air in the heat engine which emits entropy to outer space.