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Do We Feel Forces?

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ABSTRACT: Students exhibit considerable problems with mechanics. In this field we identify "misconceptions" most easily. It is argued here that we may help the learner by critically examining the question of whether or not we experience forces. An analysis shows that it is not forces we feel but the flow of momentum through matter. For example, if we hang from a cross bar, the force on us is zero but the momentum current through us is not. There is a quantity which can render precise what we feel in this situation, namely stress or momentum current density. We have a much better feeling for stress than for forces. The alternative concepts of students do not turn out to be "misconceptions" at all.

I Introduction

We commonly start teaching physics by introducing mechanics. It is argued that we have a feeling for motion and for forces which should facilitate entering an abstract field like physics. However, if we take serious recent investigations which demonstrate that students have considerable difficulties with the concept of force,[2] we have to admit that the situation cannot be so simple.

Students (1) mix up force and momentum; (2) they employ an "aristotelian" view of forces (i.e. there always has to be a force for moving an object); and (3) they cannot identify forces correctly even in simple situations[2]. This is the case even after considerable time has been spent on instructing them. I will argue that part of this problem stems from the belief of students and teachers that we have a feeling for forces which, however, is not the case. What we feel is something different, namely stress or, equivalently, the flow of momentum through an object.[3–6] Another problem has to do with the motion of bodies. We have a feeling for the thrust (or momentum) of bodies and this we call a force which contradicts theory. The learning problems should be addressed by carefully distinguishing between the flow of momentum on the one hand, and forces on the other. Force turns out to be a much more abstract concept than momentum currents. In the following sections, I will render these statements precise.

The central point is the distinction between three different types of momentum transfer. I have been speaking of momentum flow *through* bodies which is associated with stress. This type of flow is called *conductive*. The interaction of bodies and fields, however, is of a different nature. It leads to sources (and sinks) of momentum in bodies (and fields). In analogy to thermal processes, we can speak of *radiative* transfer of momentum. Due to this interaction, momentum does not flow through bodies and therefore does not lead to stress. In other words, we cannot feel the force of gravity. Finally, there are *convective* momentum currents associated with the transport of matter.

Force, it turns out, is an extremely abstract concept. In short, forces are the momentum fluxes of two of the three kinds of momentum transfer. Forces are associated with conductive and radiative momentum flow only, while stress has to do with the conductive type only. No wonder that identifying forces is a non-trivial task. Especially the fact that we exclude convective momentum currents from what we call forces is confusing. It is responsible for the trouble we have with variable mass systems.[4,7] Incidentally, the convective momentum current might well be the kind of "force of motion" which students introduce in the direction of a moving object. Naturally, approaching mechanics from this angle does not make it an "easy" subject. In fact, the road to an understanding of the concept of force is long and steep. It pays, however, to point out clearly that we do not feel forces, and that we have to reach quite a level of abstraction before we arrive at a precise understanding of their role.

In this alternative presentation of mechanics and of forces, the role of misconceptions is a different one.[3–6] This does not mean that there could not be misunderstandings. However, it seems that the alternative conceptions held by students can be used to advantage in this new approach to mechanics. We can employ our feeling for introducing the fundamental mechanical quantity: stress.

II Stress and conductive momentum currents

We can learn a lot by observing students at work when they are given the task of identifying forces. Take the example of a crate being moved across the floor at constant speed. We intuitively know that if we were in the crate's place we would be feeling "something". Something, which cannot be zero, is needed for moving the crate. Usually we call this a "force". Therefore it is quite natural to assume that a (net) force is necessary to keep the body moving (this is the aristotelian view of motion). When we study mechanics we are confused by the statement that the force on the crate is zero in this case. The same problem occurs in statics as well. I often observed students drawing only one (horizontal) force acting on a block which we press against a wall. The rationale is clear: we (and the block) feel a force; therefore, there can only be one. In general, I found that students correctly identify cases of mechanical stress. They know when and where "something is happening". Still, very often they are incapable of translating this knowledge into a sure identification of forces.

If we present mechanics using the notions of momentum (amount of motion: a substancelike quantity analogous to charge in electricity[3]) and its flow, we can address some of these problems on both an intuitive and a rational basis. We can identify the physical quantity associated with what we feel in mechanical processes, namely momentum currents through matter; these do not have to be zero even if the net force vanishes. This feeling can be translated into pictures using stream lines representing the momentum currents and their distribution, before a mathematical formalism is developed.

A simple, and nonetheless confusing, example of mechanical stress is the stretched rope (Fig.1). In the case of equilibrium, we commonly say that we pull on either end with equal but opposite forces. For many students it is not clear how large the force on the rope is. Is it zero, is it $|F|$, or is it $2|F|$? Since it is well known that we could feel something if we were at the rope's place, we tend towards the second or the third answer; however, neither is correct. The correct answer, namely zero, causes problems because it contradicts our feeling.

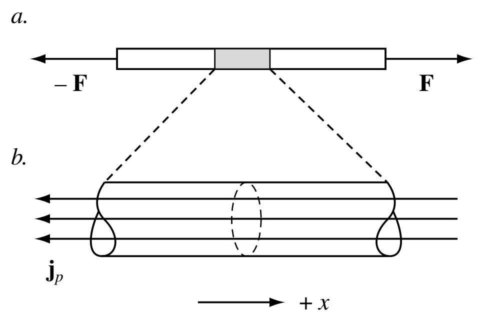

Figure 1: A rope under tension. Momentum is flowing through the rope in the negative x-direction. The momentum current per cross section (A) is the measure for the tension in the rope. $\mathbf{j}_p$ is the momentum current density.

There is a simple solution to this dilemma. The rope is under stress, and this is what we feel. We only have to make this a physically useful statement. There are ways of representing correctly what we mean with the help of the momentum current picture.[4,6] If we introduce mechanics on the basis of the concept of momentum, it is clear also for the beginner that momentum must be flowing through a stretched rope. I call the flow of momentum through the rope, which I can represent by stream lines (Fig.1 b), the cause of what the rope feels. Indeed, we can introduce a quantitative measure for our feeling, namely the amount of momentum which flows through the rope per time (the momentum current I_p) and per cross section (A). This is called the momentum current density j_p which is related to the momentum current through the rope by

$$I_p = \int_A j_p dA \quad \textbf{(1)}$$

This quantity is not equal to zero for the stretched rope, even though the net force vanishes.

The dynamical case can be developed as follows. A block is pulled by a rope across a frictionless surface (Fig.2). The block accelerates. Momentum must be supplied by the one pulling the block, and it will flow through the rope into the body. Without knowing much about mechanics, we can draw stream lines representing the distribution of momentum currents through the body. The entire momentum current enters through the point where the rope is attached (telling the future engineer that this is a point to be watched since the momentum current density is largest there). From there it flows toward the back. Constantly, some of the momentum necessary for acceleration is deposited along the path. Far from the point where momentum enters the body (the "inlet"), the current density is expected to decrease linearly (Fig.2).

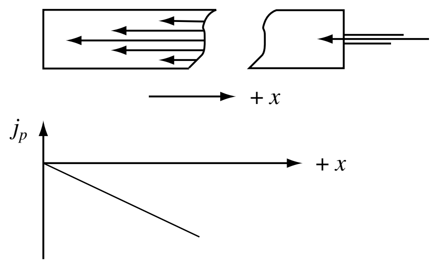

Figure 2: Momentum current distribution in a block which is accelerating to the right. Momentum accumulates in every part of the body. The magnitude of the momentum current density is largest at the right and zero at the left end of the body.

This can be stated mathematically. Take a slice of the body being accelerated (Fig.3). Assume that we are far from any "inlets" and that therefore momentum is flowing horizontally. How much momentum has to be deposited in this slice to give it a certain acceleration? At the point $x + \Delta x$ (Fig.3), momentum enters the slice from the right. There, the momentum current density is given by $j_p(x) + \Delta j_p$. Momentum is entering the body at the rate $I_p(x + \Delta x) = -A(j_p(x) + \Delta j_p)$, where A is the surface area perpendicular to the flow, and I_p is the momentum current through the surface. The rate is counted as a negative quantity because we take the orientation of the surface positive for flow out of the body. Some of this momentum will stay in the slice, the rest will flow out of it through the opposing surface at x. There, the rate at which momentum is leaving is $I_p(x) = Aj_p(x)$. This means that momentum is deposited in the slice at the rate $-A\Delta j_p$. On the other hand, this amount of momentum is used for acceleration. The time rate of change of momentum of the body is $\Delta(\rho A \Delta x\, v)/\Delta t$. Here, ρ and v are the density and the velocity of the slice, respectively. Equating the two rates leads to:

$$\rho \frac{\Delta v}{\Delta t} + \frac{\Delta j_p}{\Delta x} = 0 \qquad \textbf{(2)}$$

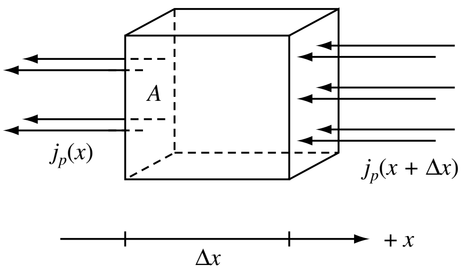

Figure 3: Momentum flows through a slice of matter of density ρ, thickness Δx, and cross section A. If the current density changes along x, momentum will be deposited in (or removed from) the slice, leading to its acceleration.

This is a simple form of the equation of motion of a (linear) continuum. If the acceleration is constant as in our second example (Fig.2), the momentum current density decreases linearly from front to back (far from the "inlet"). The solution of Eq.2 is $j_p(x) = -\rho a x$, where a is the acceleration, and x is counted from the back of the body (Fig.2). The complicated distribution of stream lines near the point where the rope is attached demonstrates that conditions must be more complicated around there. You see that qualitative ideas paired with simple mathematics allow us to treat basic continuum mechanics.

I have left out discussing the direction in which momentum flows. As in the case of electricity, we have to define the direction of flow of positive momentum. Simple rules have been given for identifying this direction.[6] In short, positive momentum flows in the positive x-direction through a compressed body, and it flows in the negative direction through matter under tension.

III Bodies in fields: Radiative transfer of momentum

I have introduced the momentum current density as the measure of what bodies feel when they are undergoing mechanical processes. However, the situation is not quite so simple; this is demonstrated by the action of gravity, inertia, and electricity, upon bodies. By studying bodies in the gravitational field we will be able to extend our treatment of continuum mechanics.

A. A first example: Free fall

Take a freely falling object. It accelerates, which means that it is receiving momentum from somewhere. We know that this momentum must be coming from the Earth. Therefore, we have momentum flowing into, and possibly through, the body via the field. However, it is well known that a person falling freely does not feel any mechanical stress. Therefore, we now are dealing with a situation in which we cannot feel the momentum current.

We can solve the problem by postulating that the momentum which is deposited in the body via the field does not flow through the object. If momentum arrives through the field directly at every part of the body, we do not have a (surface) current density j_p. In other words, we have to assume that the action of a field constitutes a source (or a sink) of momentum for the body. It has been shown elsewhere that this interpretation of gravitational and electromagnetic interactions is correct also in a mathematical sense.[4] [The actual mechanism of momentum transport through fields and through bodies in fields is much more complicated. However, the net result is the one stated here.] Therefore, it makes sense to divide momentum transports into two kinds, only one of which can be "felt" by matter, i.e., leads to stress. I shall call the second type associated with the interaction of bodies and fields *radiative transport* of momentum.

From the example of free fall we know that the gravitational field deposits momentum at every point of the body at a rate proportional to its local mass density ρ:

$$\sigma_p = \rho g \qquad \textbf{(3)}$$

This is the expected source rate density of the supply of momentum (integrated over the body, this must be equal to the net current due to the field, namely $g \cdot m$). Therefore, the equation of motion takes the form

$$\rho \frac{\Delta v}{\Delta t} + \frac{\Delta j_p}{\Delta x} = \sigma_p \qquad \textbf{(4)}$$

Here, only $\Delta j_p/\Delta x$ has to do with conductive momentum flow which is associated with stresses in the body; σ_p takes the role of a source term. In the example of free fall this means that

$$\rho g + \frac{\Delta j_p}{\Delta x} = \rho g \quad \Rightarrow \quad \frac{\Delta j_p}{\Delta x} = 0$$

Together with $j_p(0) = 0$, the solution is $j_p(x) = 0$. This is the expected result since we know that the body does not experience any stress in free fall.

B. A rope hanging from a tree

Consider this example which shows the action of both conductive and radiative momentum transports. A rope of length L, density ρ, and cross section A, is hanging from a tree (Fig.4). Our feeling tells us that the magnitude of the momentum current density (stress) must increase upward along the rope. This I first describe qualitatively using momentum stream lines and sources. Later, a simple calculation will confirm the ideas.

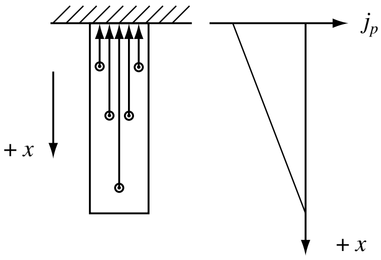

Figure 4: A rope hanging from a tree. Momentum appears in the rope via the field (circles indicate sources). It has to flow out of the rope via the point where it is attached (solid lines), leading to a linearly increasing magnitude of the conductive current density (stress). The current flows in the negative x-direction, indicating that the rope is under tension.

Momentum is deposited inside the rope via the field at the rate ρg. As I have pointed out, the body does not feel this flow. However, the momentum deposited inside the rope cannot stay there; it has to flow out via the point where it is attached to the tree. This constitutes a surface-like current through matter which is felt as stress. From the momentum stream lines we expect the current density to increase linearly upward along the rope.

This is borne out by mathematics. The equation of motion of a slice of the rope is given by (4). Since $\rho\Delta v/\Delta t = 0$, we see that

$$\frac{\Delta j_p}{\Delta x} = \sigma_p \quad \Rightarrow \quad \frac{\Delta j_p}{\Delta x} = \rho g \qquad \textbf{(5)}$$

Together with $j_p(x = L) = 0$, the solution is $j_p(x) = \rho g(x–L)$, which we had expected. [The conductive current through the rope is in the negative direction which agrees with our rule concerning the direction of momentum currents: the rope is under tension.]

C. The tides

A still more interesting example is that of a body falling in an inhomogeneous gravitational field (Fig.5). The body will experience tides. The momentum stream lines beautifully demonstrate the power of the approach taken here. Assume that the field strength g decreases linearly upward along the body. Due to this field, momentum is deposited at a higher rate in the lower portions of the body. The object accelerates at an average rate which means that the upper parts receive too little momentum while the lower parts get too much. Therefore, momentum will be redistributed throughout the body: a conductive current from the lower to the upper end is the result; since this current points in the negative x-direction (Fig.5), it tries to pull the body apart. We can also see that in the middle of our object the stress will be largest since all the momentum which is being rearranged must flow through that cross section. The conductive currents are zero at the lower and upper ends, leading us to expect a momentum current density as displayed in Fig.5.

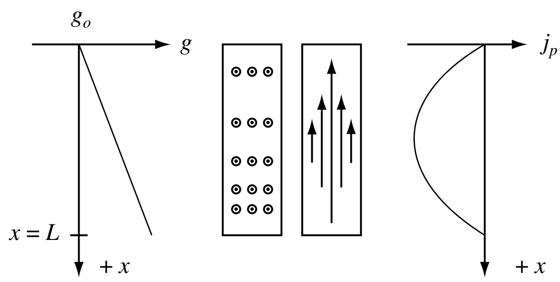

Figure 5: A body falling in an inhomogeneous field. The lower parts receive relatively too much momentum through the field. A conductive momentum current in the negative x-direction (stress) will be established so that momentum can be distributed evenly.

Again, the calculation is straight forward. Assume that g increases downward according to $g(x) = g_o + bx$ (where b = const.). The body's acceleration is $g_o + bL/2$, where L is the length of the object. The equation of motion (4) in this case is:

$$\rho\left(g_o + \frac{bL}{2}\right) + \frac{dj_p}{dx} = \rho g(x) \qquad \textbf{(6)}$$

Its solution is $j_p(x) = 1/2\rho b(x^2 – Lx)$; as expected, the current is negative in our coordinate system.

The approach can be extended without much difficulty to include hydrostatics and inertial fields.[4]

IV Convective momentum currents

Convective transport of momentum is another phenomenon we have a sort of feeling for. It is a kind of thrust, and because of this, students are often led to introduce forces in the direction of motion of bodies. Just as in the case of stress, they give the phenomenon the name "force". It is of utmost importance to point out to the learner that we are dealing with two different things which both are called force in everyday life. The problem is that in mechanics we do not call this "thrust" a force.

From the point of view of momentum transport, the distinction between the two phenomena is simple. One represents the flow of momentum through matter (without matter moving), while in the second case momentum is carried along by moving matter. We have a concrete, yet vivid, image which lets us deal with the differences encountered.

In order to understand mechanics, we need to know about the roles of conductive and convective momentum transport, respectively. The trouble with variable mass systems can be traced to the fact that we usually do not introduce the distinction between these two modes of momentum flow. This keeps us from recognizing in a simple way that forces cannot change the mass of a body. Forces only are associated with conductive and radiative momentum transports, both of which cannot transport matter across system boundaries. Therefore, it is wrong to use Newton's Second Law in any other form than $F = ma$. If we want to treat changing mass systems, we have to include convective momentum currents in the momentum balance (Eq.4). There will be another term in Newton's Law.[5,7]

V What are forces?

The identification of forces is anything but trivial. This is so because there are at least two different phenomena with which we intuitively associate forces, while the word force is reserved for still something different in physics (Table 1).

Now, what are forces in physics? There are two types. The first has to do with the conductive flow of momentum, and the second is associated with radiative momentum transport. Let me first introduce the former.

When momentum flows through matter, we describe the situation by the momentum current density (stress). Often, we are interested in the question of how much momentum flows per time through a given surface (which might be the entire surface of a body, or a part thereof, or a surface inside the body). We call this the flux of momentum. If we talk about one component of momentum only, the flux is a number which can be positive or negative, depending on the flow and the orientation of the surface. This conductive momentum flux is called a *surface force* in continuum mechanics. If the surface is the closed surface of the body, we speak of the force (or the net force) on the body. It is easily possible that the net flux is zero even though the conductive current is not zero at all. This is the source of the problem we have with the identification of forces acting on the stretched rope (Section II).

Table 1: Momentum transport and forces

Type of transport	Is stress associated with it?	Do we intuitively associate forces with it?	Do we associate forces with it in physics?
conductive	yes	yes[a]	yes
convective	no	yes[b]	no
radiative	no	no[c]	yes

a. Forces are associated with stress, not with what is called force in physics.

b. Forces are associated with the thrust of bodies.

c. Forces are introduced for the stress experienced by a body resisting at the Earth's surface, and not for the force of gravity.

In the case of the interaction of bodies and fields we are interested in how much momentum is deposited in a body per time via the field. In order to calculate this quantity, we have to integrate the source rate density of momentum over the volume of the body. The resulting radiative momentum flux is called a *body or volume force* in mechanics. We cannot feel this type of force or interaction. Still, we are intuitively ready to introduce a force associated with it, especially if the body is resting at the surface of the Earth. Very often, students only introduce this force, the rationale being that we feel something which is not zero. Adding a second force which balances the first contradicts this feeling.

As mentioned in the preceding section, students use the term "force" for the thrust they see in a moving body. This is the source of the forces introduced in the direction of motion of bodies. In physics, we cannot call this quantity a force.

VI Conclusion

It has been shown that we do not have a feeling for what physicists call force. In everyday life, the term “force” is used for other processes. The phenomena can be squared with our feelings if we accept that we only experience momentum flow through matter; flow through fields is of a different type which does not lead to stress in matter; from the viewpoint of the body it constitutes a source (or a sink) of momentum. A natural measure of what we feel is the (conductive) momentum current density. It is possible to picture what is happening by momentum stream lines. The approach directly leads to a treatment of continua rather than mass points.

Forces are abstract objects. They serve to measure the strength of momentum flow through surfaces or from fields into bodies (flux or source rate). As such they are useful for relatively simple situations known from the mechanics of mass points, and maybe that of rigid bodies. They should be introduced as what they are, namely means of accounting for momentum flow, after the role of momentum currents has been made clear. It should be kept in mind that convective momentum currents are not called forces. Introducing mechanics in such a way has considerable advantages which are worth the effort: we are given a tool for dealing with continua[4] which can be motivated directly on the basis of our everyday experience. The problems we can deal with go far beyond the mechanics of mass points.

Finally we have to ask ourselves what remains of the misconceptions which have been identified in mechanics?[2] I would say that the main misconception is the belief that we can feel forces, or to be precise, that we can feel what physicists call force. Texts on mechanics do not make it clear that such a feeling does not exist. We have a feeling for stress and for thrust (of a moving body), and these are the phenomena we associate with force in everyday life. We could circumvent some of the problems by using the word “force” for stress (we would then still have to make clear that thrust is something different). Actually, for purely didactic reasons, it would be best not to speak of forces at all. Even after my students have learned how to rationally approach the identification of forces, they return to their feelings if they are told to find those elusive arrows. And as usual, the results are catastrophic.

I can only hope that a clarification of mechanical concepts will come about by the use of the concept of momentum transport as we know it from continuum mechanics.[3,4] This should make teachers aware of the problem that misconceptions in mechanics might be of a different nature as hitherto assumed. What we identify as a misconception depends on the structure of theories taught in our classes. This holds particularly for thermodynamics.[8]

Notes and References

1 This is the slightly reworked version of the paper presented at the Second International Seminar on Misconceptions and Educational Strategies in Science and Mathematics held at Cornell University in July 1987. It was Published in the Proceedings (J. D. Novak, ed.).

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