Experiments and Models in Rolling Motion: Momentum, Angular Momentum, Energy, and Dissipation

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Here we consider rolling motion of cylinders on a horizontal soft surface leading to a gradual slowdown of motion. All cases and discussions will strictly refer to rolling motion without sliding. Experiments are performed and a dynamical model of the interaction of translational and rotational motion is constructed. In particular, we demonstrate the role of energy and dissipation in this example.

Rolling motion is easily observed but not as easily explained. Students encounter difficulties in understanding the concepts that allow an adequate quantitative description of the process of rolling without sliding. Creating a model that clearly shows forces and torques, their interactions, and energy transfers and dissipation can be a challenge.[1,2,3,4,5] For this reason, we will take pains to relate a model of the dynamics of rolling motion (Section 2) directly to concrete observations (Section 1). Actual data is used to derive the constitutive relation for a particular parameter in the proposed model (Section 2)—in this way it is hoped that the meaning of elements of the model is made clear.

While the dynamical model applies the traditional formal approach to mechanics (Section 2), the discussion in Section 3 of energy interactions in rolling motion makes use of an interpretation based upon the conceptual metaphoric network of forces of nature.[6,7,8] In recent years, the issue of conceptual metaphors has been introduced in science education research as a productive resource for learning and for conceptual change.[9,10] In a recent paper, Harrer et al.[11] discuss the importance of such resources in the field of physics. In particular, they show how use can be made of substance metaphors[12,13] for some of the basic concepts in physics. Here we demonstrate how this idea is applied to the relation between (linear) momentum, angular momentum, and energy. The image of storage and exchange of these quantities leads to a graphical model of rolling motion as a dissipative process. Forces and torques are interpreted as momentum and angular momentum flows that characterize the interaction between the body and its environment. A process diagram[14] is used to show how energy storage, energy transfer, and dissipation are related to the mechanical quantities via Mach's[15] interpretation of velocity and angular velocity as the potentials of motion. This is a generalization of Carnot's metaphor of heat as a fluid quantity flowing downhill and releasing energy (Carnot's power of heat[16]; see Fuchs[17] for a modern version of thermodynamics based upon this image).

In summary, we adopt the approach afforded by substance metaphors for quantities such as energy, momentum, angular momentum, charge, etc., which has been applied to the construction of high school and introductory university physics courses for engineers and elementary school teachers.[18,19,20,21] In these courses we make use of students' metaphoric understanding of science and apply it on a daily basis. It is important to note that our students are exposed to the concepts of storage and flow of momentum, angular momentum, and energy, velocity and angular velocity as level quantities, and the relation between level quantities, substance quantities, and energy, before we treat examples such as the one presented in this paper. We believe that our approach leads to the creation of productive images for qualitative and quantitative modeling of an instructive example.

1 Experiments

We present data of four versions of an experiment where a cylinder (C) or a cylindrical shell (S) is made to roll across a horizontal surface without sliding. The cylinders are manually set into motion to roll toward a motion sensor[22] (see Fig.1, photographs on the left) that records their velocities as functions of time (Fig.1, diagram on the right).

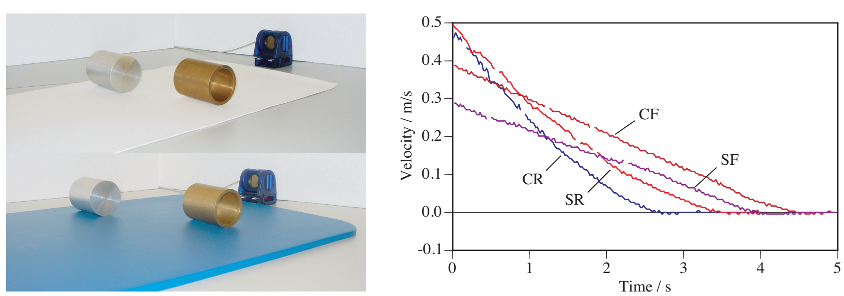

Fig. 1: *Velocities measured as functions of time for cylinder (C) or cylindrical shell (S) rolling on a sheet of thick flannel (F) or on a rubberized mat (R). Photograph at top: flannel mat; photograph at bottom: rubberized mat.*

The bodies have equal masses ($m = 1.375\,\text{kg}$) and radii ($R = 0.0400\,\text{m}$), but their shape factors K, and therefore, their moments of inertia, are different (experimental values are: $K_C = 0.50$, $K_S = 0.83$;[23] see Section 2 for the definition of shape factor). The surface is either a felt (thick flannel) sheet (F) or a rubberized gymnastics mat (R). The combinations of body and surface lead to the four experiments CF, CR, SF, and SR in Fig.1.

Data of the velocities (of the center of mass) is presented in Fig.1 (diagram on the right). We see two types of behavior depending upon which surface has been used. In the case of felt (F), the velocity decreases alsmost linearly with time until shortly before the bodies stop. In the case of rubber (R) the velocity is more noticeably a nonlinear function of time. Naturally, the bodies come to a stop after a certain timespan. If we take a close look at the motion shortly before rest, we see that the cylinders roll backwards very briefly. This suggests that the rolling bodies push a bulge of the soft mats in front of them—meaning that the contact area between cylinder and surface is not symmetrical which leads to brief backwards motion. This observation will be used to construct the particular model of forces and torques presented in Section 2 below.

2 Modeling Linear and Rotational Motion

Here we develop and discuss a consistent dynamical model that applies the traditional formal approach of introducing forces and torques (i.e., moments of forces) as elements in the equations of motion (see Fig.2).

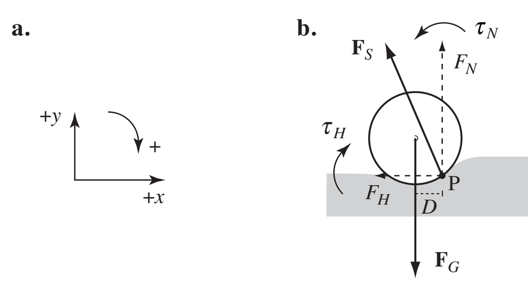

Fig. 2: *(a) Indication of choices for signs of quantities. (b) Two forces act upon the body, the force of gravity $\mathbf{F}_G$ and the force of the surface upon the body $\mathbf{F}_S$. The latter has two components, F_N and F_H. The unsymmetrical deformation of the soft surface suggests that the point of attack of $\mathbf{F}_S$ should be shifted forward in which case the torques associated with F_N and F_H lead to a consistent result for decelerated motion.*

In the $y-$direction, we need two equal but opposite forces to account for the fact that the body does not accelerate vertically. For horizontal motion, we need to introduce a retarding force. Moreover, we need a (net) torque that is negative, leading to decelerated rotational motion. If we continue with the traditional approach of modeling torques as moments of (components of) forces, we can construct a consistent representation by introducing a single contact force $\mathbf{F}_S$ having normal and a horizontal components, F_N and F_H, respectively (see Fig.2b). As we shall see in Section 3, the model allows for a consisten interpretation of energy interactions, transfers, and dissipation as well.

The observation described above in Section 1 suggests that the contact between body and (soft) surface is not symmetrical (note the rendering of the deformed mat in Fig.2b) which lets us interpret the mechanical behavior as follows. The point of attack P of $\mathbf{F}_S$ is moved to a point slightly forward to and above the lowest point. The shift forward (D) is introduced as a parameter of the formal model to be developed below. Qualitatively speaking, vertical translational motion is determined by F_G and F_N, whereas translation in the horizontal is the result of F_H. Rolling motion is influenced by the two moments of the two (components of) forces F_N and F_H, τ_N and τ_H. The condition $|\tau_N| > |\tau_H|$ needs to be satisfied if rolling motion is to be decelerated.

Now we develop the mathematical model of the dynamics of this phenomenon, i.e., we account for changes of momentum and angular momentum in terms of the action of forces and torques. To this end we need to carefully consider the geometric and kinematic relations for the rolling body with forces introduced in Fig.2b (see Fig.3).

Note that the geometry of rolling motion is governed by the fact that the velocity of the center of mass v_{CM} and angular velocity ω are strictly proportional as long as the cylinder does not slide:

$$v_{\mathrm{CM}} = R\,\omega \tag{1}$$

Since $p_x = m\,v_{\mathrm{CM}}$ and $L = J\,\omega$, where

$$J = K\,m\,R^2 \tag{2}$$

is the moment of inertia and K is the *shape factor* of the rolling body, Eq.(1) leads to $L = J\,p_x/\,(m\,R)$ or $L = K\,R\,p_x$. This also holds for the rates of change of momentum and angular momentum giving us the time independent relationship:

$$\dot{L}/\dot{p}_x = K\,R \tag{3}$$

Equations of motion are delivered by the balance of momentum and of angular momentum of the rolling body:

$$\dot{p}_x = F_H \tag{4}$$

$$\dot{L} = \tau_N + \tau_H \tag{5}$$

For signs of forces and torques, see Fig.2b. To make use of the laws of motion, we need expressions for forces and torques. Since we model the torques as moments of forces, and since we refer rotational motion to the center of mass CM of the cylinders, we have:

$$\tau_H = -A\,R\,F_H \tag{6}$$

$$\tau_N = -D\,F_N \tag{7}$$

where

$$A = \sqrt{1 - D^2/R^2} \tag{8}$$

In our experiments (Fig.1), A turns out to be alsmost exactly equal to 1; it is smaller than 1 by typically less than a tenth of one percent.

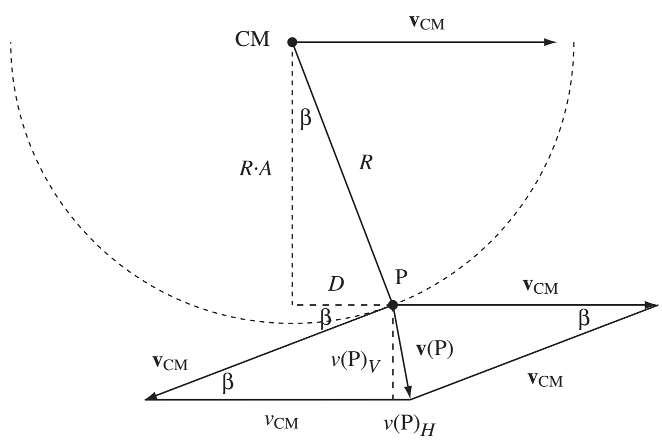

Fig. 3: *Geometrical relations and motion of the point of contact of the force* $\mathbf{F}_S$ *relative to a stationary observer.*

At first sight it looks like there are two independent parameters of the motion: the horizontal component of $\mathbf{F}_S$, i.e., F_H, and the displacement D (see Fig. 2). However, in our model of rolling without sliding, the rigid form of coupling of translation and rotation relates one of these to the other:

$$\frac{F_H}{F_N} = -\frac{D}{(K + A)\,R} \tag{9}$$

so that there is only one free parameter, which we will choose to be D. Equivalently, the ratio of the torques describing the exchange of angular momentum, with $|\tau_N|$ always greater than $|\tau_H|$, is given by:

$$\frac{\tau_N}{\tau_H} = -\left(\frac{K}{A} + 1\right) \tag{10}$$

Now, since the magnitude of F_N equals $m\,g$, and $F_H = m\,\dot{v}_{\mathrm{CM}}$, we can use Eq.(9) in order to determine the parameter D from the observed behavior $v\,(t)$ in Fig.1. Since A is very close to 1, we can simplify the quadratic equation and obtain

$$D = -\frac{(1 + K)\,R}{g}\dot{v}_{\mathrm{CM}} \tag{11}$$

If the model of unsymmetrical contact between rolling body and soft surface with the forward shift of the point of attack of F_S is to be realistic, we should expect a couple of results. First and foremost, D should be the same for the solid cylinder (C) and the cylindrical shell (S) rolling on the same mat—radius and weight of the bodies are equal and we should not expect the soft surface to "feel" the difference of internal distribution of mass. Second, D should be strongly nonlinear for the rubber mat (R) and much less so for felt (F). Third, D is expected to be noticeably smaller for felt than for rubber. Fig.4 summarizes the results of an analysis of data in Fig.1—as we can see, all three expectations are fulfilled. In other words, we can express the constitutive relation for motion upon a particular surface as a particular function $D\,(v_{\mathrm{CM}})$, at least to a sufficient degree.

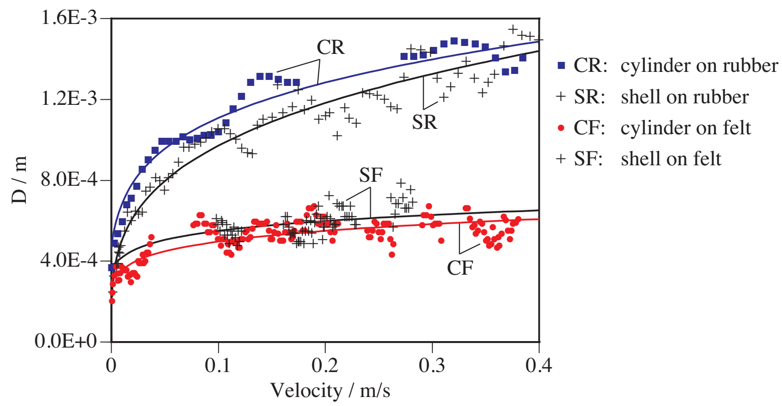

Fig. 4: *Determination of the parameter D as a function of v_{CM} from data in Fig.1. Analysis is based upon Eq.(11). Solid symbols are for the cylinder, crosses are for the cylindrical shell used in the experiments. The original data sets have been cleaned up and smoothed using moving averages to obtain the data shown here. Note that the results for D are close for experiments* CR *and* SR *on the one hand, and* CF *and* SF *on the other. The fitting functions used here are potential functions.*

In summary, the differential equation for horizontal motion can be formulated in terms of the rate of change of the velocity of the center of mass and the constitutive relation $D\,(v_{\mathrm{CM}})$. The equations of balance of momentum p_x, i.e., $m\,\dot{v}_{CM} = F_H$, leads to

$$\dot{v}_{\mathrm{CM}} = -\frac{g}{(K+1)\,R} D\,(v_{\mathrm{CM}}) \qquad (12)$$

Here we have set $A = 1$. If rolling upon a selected surface is modeled in terms of a particular power function $D\,(v_{\mathrm{CM}}) = a\,v_{\mathrm{CM}}^b$ in a dynamical model of the observed motion, we obtain very satisfactory results. The parameters a and b of the potential function obtained by fitting simulations to data sets are given in Table 1. If we accept the relation $D\,(v_{\mathrm{CM}}) = a\,v_{\mathrm{CM}}^b$, the solution of Eq.(12)

becomes

$$v(t) = \left[v_0^{1-b} - a(1-b)\,\frac{g}{(K+1)\,R}\,t \right]^{1/(1-b)} \tag{13}$$

Tab. 1: *Parameters for rolling upon different surfaces*

$D(v_{\mathrm{CM}}) = a\,v_{\mathrm{CM}}^{b}$	a	b
Rubber mat, shell	$1.78 \cdot 10^{-3}$	$2.55 \cdot 10^{-1}$
Rubber mat, cylinder	$1.87 \cdot 10^{-3}$	$2.15 \cdot 10^{-1}$
Felt mat	$5.75 \cdot 10^{-4}$	$2.44 \cdot 10^{-2}$

3 Energy Interactions and Dissipation in Rolling Motion

Now we turn to a discussion of what energy is doing in all of this. Our presentation rests upon the type of metaphors and the tools derived from them discussed already in the Introduction. First and foremost, we need to graphically interpret momentum and angular momentum as we would fluids, electricity, or heat—namely, as fluidlike quantities.[24] They are stored in moving bodies, and they are exchanged with the environment in mechanical interactions.[25]

Velocity and angular velocity are the potentials associated with momentum and angular momentum, respectively. Therefore, when momentum or angular momentum flow from a point of higher to lower potential, energy is released at the rate

$$\mathcal{P} = -\triangle\varphi_X\ |I_X| \tag{14}$$

where $\mathcal{P}$ is the power of the process, φ is the potential, and I is the current of either momentum or angular momentum. The index X denotes either translational or rotational motion. If momentum or angular momentum go from points of lower to points of higher potential, energy is required at the rate given by Eq.(14)—we say that the fluid quantities are pumped. We borrow this image from Sadi Carnot's metaphoric interpretation of the operation of heat engines.[26] As a corollary, when momentum enters or leaves a body at a point where (a component of) the velocity of the body equals v, there is an energy current

$$I_{E,i} = v\,I_{p,i} \tag{15}$$

entering or leaving the body. $I_{p,i}$ is a component of the momentum current vector which is identical to the component of the force we associate with the particular momentum transfer. We can interpret this equation figuratively by saying that momentum acts as the carrier of energy in translational processes.[27]

Having said this, we are ready to create a graphic representation of energy interactions, transfer, and dissipation for our example of rolling motion without slipping upon a horizontal surface (Fig.5).

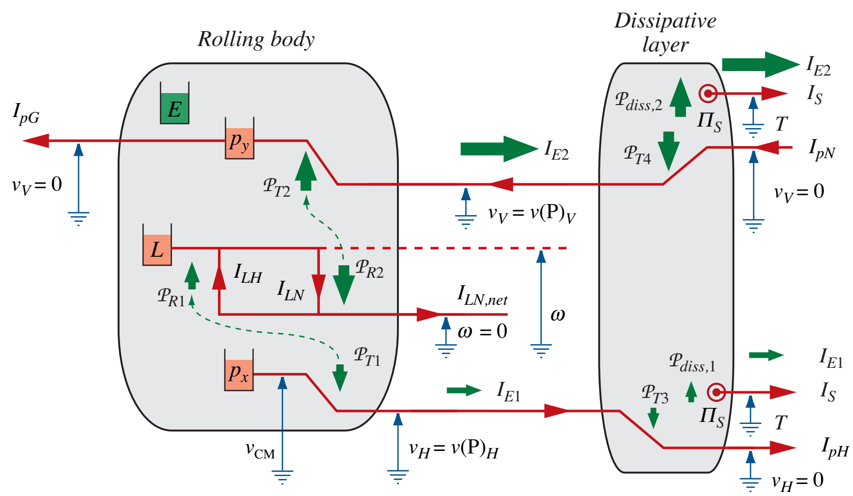

Fig. 5: *A process diagram for the rolling body and a dissipative layer (deforming layer between a rigid body and a rigid surface). Container symbols are for stored quantities. Momentum for the $x-$direction leaves the rolling body; $y-$momentum enters the body; angular momentum both enters and leaves. Fat vertical arrows indicate rates of energy released and used (power). The energy accompanying the outflow of momentum is dissipated in the dissipative layer and entropy is produced (circle with central dot: enropy production rate). Relative sizes of the fat arrows for power and for energy flows are an indication of relative magnitues of these quantities.*

The description goes as follows. If we start with the horizontal component of momentum, we see that it leaves the body at the point P in Fig.3. The speed of P in the horizontal direction equals

$$v\,(P)_H = (1 - A)\,v_{\mathrm{CM}} \tag{16}$$

Therefore, we have energy leaving the rolling body at a rate

$$I_{E1} = v\,(P)_H\,I_{pH} \tag{17}$$

where I_{pH} equals the horizontal component of $\mathbf{F}_S$: $I_{pH} = F_H$. This is a very small fraction of the rate at which the energy of the body changes due to the change of p_x. This observation is interpreted as follows: as momentum p_x flows through the body to the surface, it falls from points of higher to points of lower (horizontal) speed; energy is released at the rate

$$\mathcal{P}_{T1} = -\left(v\,(P)_H - v_{\mathrm{CM}}\right)|I_{pH}| \tag{18}$$

This energy is ready to be used in the process of pumping angular momentum into the body at a rate that is equal to the torque τ_H which is interpreted as the angular momentum current I_{LH}. In other words, $\mathcal{P}_{R1}$ should be equal to $\mathcal{P}_{T1}$:

$$\begin{aligned}\mathcal{P}_{T1} &= -\left(v\left(P\right)_H - v_{\mathrm{CM}}\right)\left|I_{pH}\right| \\ &= -\left[\left(1-A\right)v_{\mathrm{CM}} - v_{\mathrm{CM}}\right]\frac{D\,m\,g}{R\left(K+A\right)} \\ &= A\,v_{\mathrm{CM}}\frac{D\,m\,g}{R\left(K+A\right)}\end{aligned}$$

$$\begin{aligned}\mathcal{P}_{R1} &= -\left(\omega - 0\right)\left|I_{LH}\right| \\ &= -\frac{v_{\mathrm{CM}}}{R}R\,A\,\left|F_H\right| = -A\,v_{\mathrm{CM}}\frac{D\,m\,g}{R\left(K+A\right)}\end{aligned}$$

The story continues as follows. There is angular momentum leaving the body at a rate that equals the torque τ_N which is interpreted as the angular momentum current I_{LN}. The angular momentum falls from a level of ω to 0 whereupon it releases energy at the rate

$$\mathcal{P}_{R2} = -\left(0-\omega\right)\left|I_{LN}\right| \tag{19}$$

The energy is not communicated to the environment; rather, it is needed to pump vertical momentum at the rate $I_{pN} = F_N$ into the body; momentum goes from the negative vertical speed of point P, i.e.,

$$v\left(\mathrm{P}\right)_V = -\frac{D}{R}v_{\mathrm{CM}} \tag{20}$$

to a level of zero. $\mathcal{P}_{R2}$ should equal $\mathcal{P}_{T2}$:

$$\begin{aligned}\mathcal{P}_{R2} &= -\left(0-\omega\right)\left|I_{LN}\right| \\ &= \frac{v_{\mathrm{CM}}}{R}D\,F_N = \frac{v_{\mathrm{CM}}}{R}D\,m\,g\end{aligned}$$

$$\begin{aligned}\mathcal{P}_{T2} &= -\left(0 - v\left(\mathrm{P}\right)_V\right)\left|F_N\right| \\ &= -\frac{D}{R}v_{\mathrm{CM}}\,m\,g\end{aligned}$$

Figuratively speaking, the coupling between momentum and angular momentum transports shifts energy from translation to rotation and vice-versa. Energy transfers between body and environment take place only as the result of momentum transfers to and from the environment, i.e., I_{E1} and I_{E2} in Fig.5, at point P that is moving at velocity $\mathbf{v}\left(\mathrm{P}\right)$. The former was calculated above in Eq.(17); the later equals

$$I_{E2} = v\left(\mathrm{P}\right)_V\,I_{pN} \tag{21}$$

Summing up, the total rate at which energy leaves the rolling body equals

$$\begin{aligned} I_{E1} + I_{E2} &= v\,(P)_H\, I_{pH} + v\,(\mathrm{P})_V\, I_{pN} \\ &= (1 - A)\, v_{\mathrm{CM}} F_H + \left(-\frac{D}{R} v_{\mathrm{CM}}\right)\left(-\frac{R\,(K + A)}{D} F_H\right) \\ &= (K + 1)\, v_{\mathrm{CM}} F_H \end{aligned}$$

or

$$I_{E,total} = -\,(K + 1)\, m\, v_{\mathrm{CM}} \dot{v}_{\mathrm{CM}} \qquad (22)$$

If we integrate the outflow of energy over the time span of the entire rolling process, we obtain

$$E_{tr} = -\frac{1}{2}\,(K + 1)\, m\, v_{\mathrm{CM}}^2 \qquad (23)$$

which equals the total initial energy (of translational and rotational) motion the body possesses at the beginning of the rolling motion. This amount of energy is dissipated in the dissipation layer that is part of the model in Fig.5. Physically, the dissipation layer is made up of the soft mats used in the experiment. Obviously, for the energy released in the layer, i.e., $\mathcal{P}_{T3} + \mathcal{P}_{T4}$, to be dissipated, the material needs to respond to the flow of momentum as a viscoplastic material would—there is internal friction which leads to the production of entropy.

4 Conclusion

In this paper we have presented concrete data of the horizontal rolling motion (without sliding) of two cylinders on different soft surfaces—the materials used are a thick felt sheet and a rubber mat. Velocities of the center of mass of the rolling cylinders (a solid cylinder and a thick cylindrical shell having the same masses and radia) have been recorded as functions of time. As expected, the velocities are decreasing with time—the bodies come to rest eventually.

Data can be used to determine the one free parameter of the model we construct—the distance D by which the force of the surface upon the body is shifted forward relative to the center of mass of the cylinder. Both cylinders apparently deform a particular surface almost identically leading to the same values of the parameter D. It turns out that D is close to constant for the felt surface for much of the motion whereas it is noticeably variable in the case of the rubber mat. In all four cases recorded, D may be made dependent upon speed, and a proposal for a simple constitutive relation $D\,(v)$ is made.

In the second part of the paper we have demonstrated the source of dissipation of the breaking motion. We could show that the flow of momentum from the body through the surface and into the table (i.e., the flow related to the contact force) is dissipative—entropy is produced. If the rolling body is assumed to be rigid, dissipation is taking place in a dissipation layer made up of a viscoplastically deformable part of the mat compressed by the rolling cylinder. The energy dissipated is the energy released by the transport of momentum out of the body and “falling” from a level equal to the velocity of the point of attack of the contact force to a level of zero (velocity of the hard surface of the table). Inside the body, there are two instances of energy interactions between translation and rotation—energy released by horizontal momentum is used to pump angular momentum, and angular momentum falling from higher to lower angular velocity releases energy that is used to pump the vertical component of momentum (which is then flowing off due to the action of the gravitational field, yielding the observed balance in the vertical direction).

Notes

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[14] See Endnote 6, Chapter 2.

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[17] See Endnote 6.

[18] T. Corridoni, M. D'Anna, H. U. Fuchs, “Damped Mechanical Oscillator: Experiment and Detailed Energy Analysis” *The Physics Teacher* **52**, 88-90 (2014).

[19] See Endnote 6.

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[22] In our experiments, we used a PASCO Motion sensor PS-2103A and data was collected and elaborated with Capstone software (http://www.pasco.com).

[23] These values are measured in a separate experiment. The cylinders are mounted so that they can rotate around a fixed axis. They are made to rotate by a known torque and the angular acceleration is measured. From this the moments of inertia can be determined.

[24] See Endnotes 6 and 18.

[25] Since momentum and angular momentum are vectorial quantities, we need to treat their components separately as fluid quantities. In our case, we have to deal with $x-$ and $y-$momentum, and with one component of angular momentum. Values of these fluidlike quantities are allowed to be positive and negative. Each of these quantities satisfies a conservation requirement.

[26] See Endnotes 16 and 6.

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Tab. 2: *Quantities and units*

Symbol	***Unit***	***Explanation***
A		$A = \sqrt{1 - D^2/R^2}$
a	m	Parameter in constitutive expression for shift D
b		Exponent in constitutive expression for shift D
D	m	Forward shift of contact force $\mathbf{F}_S$
E	J	Energy content of a body
E_{tr}	J	Amount of energy transferred (work)
$\mathbf{F}_G$	N	Force of gravity
$\mathbf{F}_S$	N	Contact force between rolling body and surface
F_H	N	Horizontal component of $\mathbf{F}_S$
F_N	N	Normal component of $\mathbf{F}_S$
I_E	W	Energy current
I_L	N·m	Angular momentum current (equal to torque)
I_p	N	Momentum current (equal to force)
I_S	W/K	Entropy current
J	kg·m^2	Moment of inertia
K		Shape factor of cylinders ($J = K\, m\, R^2$)
L	N·m·s	Angular momentum
m	kg	Mass
R	m	Radius of cylinders
p	N·s	Momentum
$\mathcal{P}$	W	Power of a process
$\mathcal{P}_{diss}$	W	Dissipation rate
φ		Potential
Π_S	W/K	Entropy production rate
τ_H	N·m	Torque (moment of component of force F_H
τ_N	N·m	Torque (moment of component of force F_N
v_{CM}	m/s	Velocity of center of mass (CM)
ω	1/s	Angular velocity