Second Law Analysis of an Air-Cooled Solar Collector

Hans U. Fuchs
Department of Physics and Mathematics
Zurich University of Applied Sciences, Winterthur
8401 Winterthur, Switzerland

Hans.Fuchs@zhwin.ch

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ABSTRACT: This paper presents an example of the application of a theory of the dynamics of heat to solar thermal processes. It shows how we can deal with dynamical processes rather than equilibrium states in a straight forward manner. The challenge of applications in engineering and the sciences is met with a simplified version of continuum thermodynamics applicable to introductory physics and engineering courses.

I Introduction

Classical thermodynamics is a theory of the statics of heat. According to Callen[1], “The single, all-encompassing problem of thermodynamics is the determination of the equilibrium state that eventually results after the removal of internal constraints in a closed, composite system.”

Imagine this statement to be made about mechanics, and engineers being allowed to compute only equilibrium states. Naturally, practical thermodynamics is long past this state, but the teaching of thermal physics still gives the impression that equilibrium is all we can hope for. We have to contend with a type of mathematics unknown to any other branch of physics, and with “heat transfer” taught separate from “thermodynamics.”[2]

Modern applications of thermodynamics—for example in solar energy engineering[3]—paint a different picture. There we need to compute dynamical processes, not equilibrium states. Thermodynamics and heat transfer are unified, and time appears explicitly in our equations. The Second Law has become a central tool for analysis, and processes are judged and optimized on the basis of irreversibility (entropy production).[4]

A modern version of thermodynamics exists in continuum physics.[5] Recently, we have transferred the approach of the continuum theories to introductory engineering thermodynamics[6] and to introductory college physics.[7-9] We take entropy as the basic thermal quantity and formulate a law of balance for it. This creates the most general form of the Second Law. Temperature takes the role of the thermal potential. Aided by constitutive laws for entropy fluxes and entropy production rates, we compute processes. The energy principle is used to provide much needed additional information for constitutive theories.

In this paper we will present an example of the analysis of solar processes. We will investigate a simple model of an air cooled collector which demonstrates the importance of Second Law analysis in thermal design and optimization (Section II). The example makes use of the standard approach of modern engineering thermodynamics. Temperatures and energy fluxes are computed on the basis of the First Law. Then the overall rate of entropy production is calculated for the system, and the condition of minimal irreversibility is determined.

In Section III we will demonstrate a direct approach to the same problem based on a theory of the dynamics of heat[6] which employes entropy and temperature as the fundamental quantities. We briefly discuss how entropy can be introduced from the start and show how entropy production rates can be calculated for the various irreversible processes taking place in the collector and the surroundings. Using the laws for entropy storage, entropy fluxes and entropy production rates, we can calculate all pertinent quantities based on the Second Law alone.

II Optimal design of a solar collector

We want to know if an air-cooled solar collector should be built long and narrow or short and wide. For practical purposes, this question asks whether we should connect a number of panels in series or in parallel to get an optimal effect.[10]

To build a model, we shall consider a thin and wide rectangular duct through which air is pumped. The upper side of the duct serves as the absorber of solar radiation. The lower side is perfectly insulated. The collector shall be operated in such a way as to deliver air at a prescribed fixed outlet temperature for fixed given inlet temperature. This means we have to adjust the flux of mass of air through the collector when the length to width ratio is changed.

The air will be treated as an ideal gas having constant heat capacities. We will consider steady-state conditions in our models unless otherwise stated. The models will be single-node spatially homogenous representations of the systems. This means, for example, that the temperature of the air in the collector has a single value which is taken to be equal to the outlet temperature.

A. The balance of energy

Performing a steady-state balance of energy on the absorber and the air in the duct lets us relate temperatures and energy fluxes. The energy fluxes with respect to the absorber and the air are shown in Fig.1.

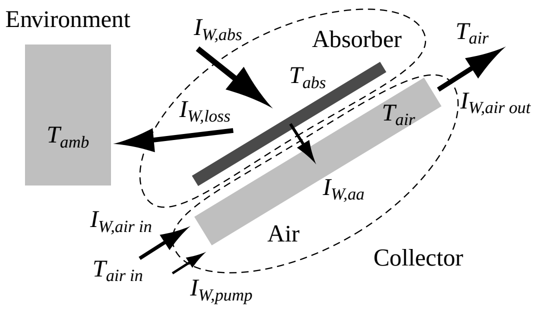

Figure 1: An air-cooled solar collector is divided into two separate systems—the absorber and the air. The figure shows the energy fluxes with respect to the systems. I_W denotes an energy flux whereas T denotes temperatures.

The steady-state form of the first law expresses the idea that the sum of all energy fluxes I_W (measured in Watts) with respect to a system must be zero. Here we have two systems—absorber and air. There are three energy fluxes with respect to the absorber: the rate of absorption of energy $I_{W,abs}$, the flux due to heat loss $I_{W,loss}$, and the rate of transfer of energy to the air $I_{W,aa}$. For this system the First Law states that

$$0 = I_{W,abs} - I_{W,loss} - I_{W,aa} \qquad (1)$$

In the case of the air it takes the form

$$0 = I_{W,aa} + I_{W,air\,in} - I_{W,air\,out} + I_{W,pump} \qquad (2)$$

Here, $I_{W,air\ in}$ and $I_{W,air\ out}$ represent the convective energy fluxes carried with the air entering and leaving the collector. $I_{W,pump}$ is equal to the pumping power.

B. Constitutive laws for the energy fluxes

We need constitutive expression for the energy fluxes to make use of the First Law.[11] The rate of absorption of energy is commonly expressed as a fraction ($\tau\alpha$) of the insolation which is the product of the irradiance G and the surface area A of the absorber:

$$I_{W,abs} = (\tau\alpha)AG \qquad (3)$$

($\tau\alpha$) is called the transmission-absorption factor of the absorber (including glass covers). The loss of the collector to the environment and the rate of transfer to the fluid in the duct are written in terms of the temperature differences and an energy conductance which is the product of a heat transfer coefficient and the surface area:

$$I_{W,loss} = AU_L\left(T_{abs} - T_{amb}\right) \qquad (4)$$

$$I_{W,aa} = AU_{aa}\left(T_{abs} - T_{air}\right) \qquad (5)$$

where

$$U_{aa} = U_{aa,o} + kv \qquad (6)$$

U_L and U_{aa} are the heat transfer coefficients due to loss and transfer to the fluid, respectively. The latter is a linearly increasing function of fluid speed v in our model. To obtain the desired outlet temperature for given fixed inlet temperature, the fluid speed has to increase as the collector is made longer.

In Eq.2, we have the net convective flux due to energy transport with the fluid. For an ideal gas this is

$$I_{W,air\,in} - I_{W,air\,out} = c_p I_m \ln\left(\frac{T_{air\,in}}{T_{air}}\right) \qquad (7)$$

c_p and I_m are the specific heat at constant pressure and the mass flux of the air, respectively. We still need to calculate the pumping power. It is determined from a model of turbulent flow through the rectangular duct. The output of the model is the pressure drop as a function of fluid speed. From the pressure drop Δp and the mass flux I_m we can calculate the pumping power:

$$I_{W,pump} = \Delta p \frac{I_m}{\rho} \qquad (8)$$

ρ is the density of the air. The pumping power increases strongly as the collector is made longer and less wide.

The equations presented so far suffice to compute temperatures and energy fluxes. The total convective energy flux $I_{W,conv} = I_{W,air\ out} - I_{W,air\ in}$ and the net gain $I_{W,conv} - I_{W,pump}$ are of particular interest. As demonstrated in Fig.2, they both increase monotonically as a function of the length of the collector. There is no indication of an optimal value of the length of the collector on the basis of energy quantities.

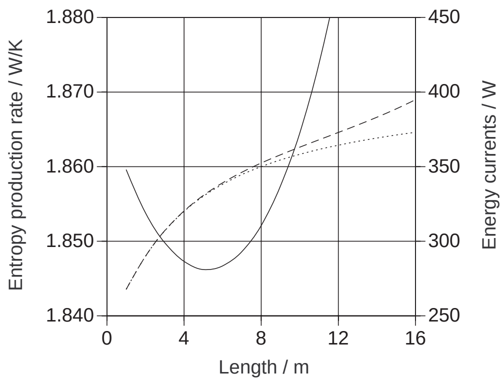

Figure 2: Total convective flux (dashed line), energy gain (dotted line), and entropy production rate (solid line) of the system as a function of length of the collector. The results have been calculated for constant irradiance and for a surface area of one square meter.

C. The balance of entropy

Optimal thermal design is based on the condition of minimal irreversibility, i.e., minimal entropy production.[4,6] Sometimes, minimal entropy production and optimal values of energy quantities coincide, but in our example they do not. This makes a Second Law analysis all the more important.

Use of the Second Law including entropy production minimization is not very well known in physics, certainly not in introductory college physics. Therefore, more information regarding entropy will be given below in Section III. Here, we will briefly present the analysis as it is known in modern engineering thermodynamics.

The Second Law is used to calculate the rate of production of entropy of a system. As discussed in Section III, in the steady-state, all the entropy fluxes I_S and the rate of entropy production Π_S in the system must add up to zero (Fig.3):

$$0 = I_{S,rad} + I_{S,air\,in} - I_{S,air\,out} - I_{S,loss} + \Pi_S \tag{9}$$

Entropy fluxes and entropy production rates are measured in W/K. If constitutive expressions are introduced into this equation we have

$$\Pi_S = -\frac{4}{3}\frac{(\tau\alpha)AG}{T_{sun}} + \left[c_p \ln\left(\frac{T_{air}}{T_{air\,in}}\right) - \frac{R}{M_o}\ln\left(\frac{p_{air}}{p_{air\,in}}\right)\right] I_m - \frac{I_{W,loss}}{T_a} \tag{10}$$

R is the universal gas constant and M_o is the molar mass of the ideal gas. The first term represents the entropy flux carried by the radiation of the sun, the second is the entropy current carried by the air, and the third equals the entropy current into the environment (see Section III). The entropy production rate can now be calculated for the steady-state processes undergone by our collector (see Fig.2). The result demonstrates that there exists a condition for optimal thermal design: the entropy production rate has a minimum at a certain value of the length of the collector.

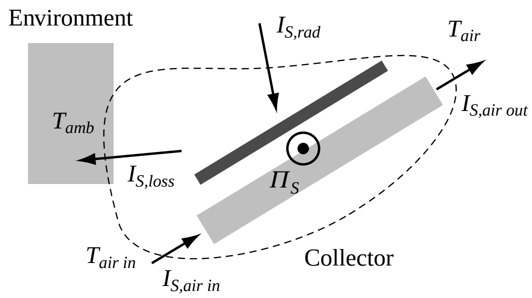

Figure 3: The system used to perform the Second Law analysis includes the collector and a part of the environment. This ensures that all relevant contributions to the entropy production are part of the system.

III A direct approach to the Second Law

In this section we will briefly discuss what is needed to employ a direct approach to thermal phenomena using the Second Law. Finally we will revisit the example of the air-cooled collector from this new viewpoint.

A. Introducing entropy and the Second Law

Entropy is introduced in analogy to charge in electricity or momentum in mechanics.[12] It is the thermal quantity which is responsible for making a stone warmer or for melting a piece of ice. It flows into and out of bodies, and it is stored there. In contrast to charge or momentum it is not conserved. It is created in irreversible processes, but it cannot be destroyed. If we introduce formal quantities for these ideas—fluxes I_S for the transports, production rates Π_S for the creation of entropy, and the entropy function S for the entropy stored in a body—we can formulate a law of balance just as we do for charge or momentum:

$$\dot{S} = \sum_i I_{Si} + \Pi_S \tag{11}$$

This is the integral form of the law of balance of entropy for a body. It is used like Newton's law of motion. If we can find the constitutive laws for the quantities appearing in Eq.11 we can determine the functions of time which describe the thermal evolution of the system.

To find thermal constitutive laws we make use of the energy principle and the relation of entropy, temperature and energy. Carnot demonstrated how this can be done.[13] His comparison of entropy and water as they drive heat engines and water turbines, respectively, motivates the tool we are going to use below. If an entropy current I_S flows from a point at temperature T_1 to a point at lower temperature T_2, energy is released at the rate

$$P_{th} = (T_1 - T_2) I_S \tag{12}$$

This quantity we call *thermal power* (Carnot's *puissance du feu*). At the same time, when entropy enters a body at temperature T, it is accompanied by a flux of energy according to

$$I_W = T I_S \tag{13}$$

This is the energy current in heating and in cooling.[14] Using Carnot's approach to thermal physics has been described before in this journal[15] and in much more detail in Chapter 1 of *The Dynamics of Heat*.[6]

B. Entropy production rates

We begin by looking at an immersion heater running at steady state. The entropy emitted is equal to the entropy produced which means that

$$0 = I_S + \Pi_S \tag{14}$$

The law of balance of energy for the immersion heater tells us that the rate of dissipation of energy $\mathcal{D}$ and the energy flux I_W emitted together with entropy add up to zero as well:

$$0 = \mathcal{D} + I_W \tag{15}$$

If we combine these two expression with Eq.13, we obtain the following result:

$$\mathcal{D} = T\Pi_S \tag{16}$$

It is more general than the derivation suggests. Whenever energy is dissipated at a rate $\mathcal{D}$ at temperature T, entropy is produced at the rate given by Eq.16.

Let us turn to overall heat transfer. Energy enters and leaves a slab of material. Entropy enters and leaves as well, and more entropy is produced in the material. Combining the steady-state versions of the laws of balance of entropy and of energy with Eq.13 yields

$$\Pi_S = T_1 I_{S1}\left(\frac{1}{T_2} - \frac{1}{T_1}\right) \tag{17}$$

for the rate of production of entropy in overall heat transfer. The index 1 refers to the point of higher temperature where entropy and energy enter the slab.[16]

To calculate the rate of production of entropy as a result of the absorption of black-body radiation we need detailed thermodynamic information about radiation.[17] Here we will only present the result:

$$\Pi_S = \left(\frac{1}{T} - \frac{4}{3}\frac{1}{T_r}\right)I_W \tag{18}$$

I_W represents the rate of absorption of energy of radiation, and T_r denotes the temperature of this radiation.[18]

The last important example for our purpose is the mixing of a stream of ideal gas of temperature T_1 with the same ideal gas in a tank having temperature T_2. Consider a system containing the gas at T_2. Gas with a mass flux I_m is entering at T_1. Let an equal mass flux be emitted at T_2. To obtain thermal steady-state operation as well, the system is heated with a current of entropy I_S. The laws of balance of entropy and energy are

$$0 = I_S + I_{S.conv} + \Pi_S \tag{19}$$

$$0 = I_W + I_{W,conv} \tag{20}$$

The constitutive laws for the fluxes are

$$I_W = T_2 I_S \tag{21}$$

$$I_{S,conv} = \left(-\frac{R}{M_o}\ln\left(\frac{p_1}{p_2}\right) + c_p \ln\left(\frac{T_1}{T_2}\right)\right)I_m \tag{22}$$

$$I_{W,conv} = c_p\left(T_1 - T_2\right)I_m \tag{23}$$

If we combine the equations we obtain the following expression for the mixing of ideal gases at different temperatures:

$$\Pi_S = I_m\left[\frac{R}{M_o}\ln\left(\frac{p_1}{p_2}\right) + c_p\left(\frac{1}{T_2}\left(T_1 - T_2\right) - \ln\left(\frac{T_1}{T_2}\right)\right)\right] \tag{24}$$

In the following discussion of our model of the air-cooled collector we will use the special expressions for entropy production rates presented here.

C. Revisiting the model

We now employ the direct approach to model the collector. To do so we have to consider all the processes taking place and determine both entropy transfer and irreversibilities. The processes taking place are (1) absorption of radiation, (2) entropy loss to the environment due to convection and radiation, (3) entropy transfer from the absorber to the fluid, (4) convective entropy transfer and mixing and (5) fluid friction. As a result we have five sources of entropy production (Fig.4).

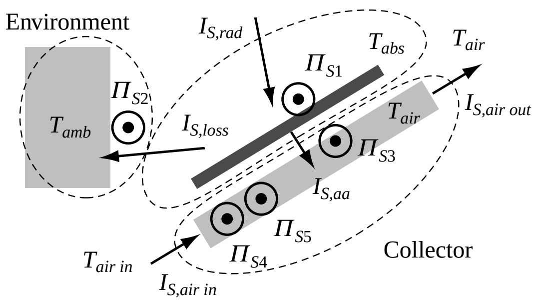

Figure 4: Entropy transfers and entropy production in the model of the air-cooled solar collector. There are five processes which all are irreversible. Four of these are associated with entropy transfers and mixing, the fifth is due to fluid friction.

The laws of balance of entropy for the absorber and the air in the duct take the forms

$$0 = I_{S,rad} - I_{S.aa} - I_{S,loss} + \Pi_{S1} \tag{25}$$

$$0 = I_{S,aa} - I_{S,conv} + \Pi_{S3} + \Pi_{S4} + \Pi_{S5} \tag{26}$$

In Eq.26, $I_{S,conv}$ denotes the net convective entropy current as in Eq.22. The fluxes appearing in the laws of balance are given by the following constitutive expressions:

$$I_{S,rad} = \frac{4}{3}\frac{(\tau\alpha)G}{T_{sun}} \quad \textbf{(27)}$$

$$I_{S,aa} = \frac{AU_{aa}}{T_{abs}}\left(T_{abs} - T_{air}\right) \quad \textbf{(28)}$$

$$I_{S,loss} = \frac{AU_L}{T_{abs}}\left(T_{abs} - T_{amb}\right) \quad \textbf{(29)}$$

$$I_{S.conv} = \left(-\frac{R}{M_o}\ln\left(\frac{p_{air}}{p_{air\,in}}\right) + c_p \ln\left(\frac{T_{air}}{T_{air\,in}}\right)\right)I_m \quad \textbf{(30)}$$

Finally we determine the five entropy production rates and their sum:

$$\Pi_{S1} = \left(\frac{1}{T_{abs}} - \frac{4}{3}\frac{1}{T_{sun}}\right)I_{W,abs} \quad \textbf{(31)}$$

$$\Pi_{S2} = T_{abs} I_{S,loss}\left(\frac{1}{T_{amb}} - \frac{1}{T_{abs}}\right) \quad \textbf{(32)}$$

$$\Pi_{S3} = T_{abs} I_{S,aa}\left(\frac{1}{T_{air}} - \frac{1}{T_{abs}}\right) \quad \textbf{(33)}$$

$$\Pi_{S4} = I_m \frac{R}{M_o}\ln\left(\frac{p_{air\,in}}{p_{air}}\right) + c_p I_m\left(\frac{1}{T_{air}}\left(T_{air\,in} - T_{air}\right) - \ln\left(\frac{T_{air\,in}}{T_{air}}\right)\right) \quad \textbf{(34)}$$

$$\Pi_{S5} = \frac{I_{W,pump}}{T_{air}} \quad \textbf{(35)}$$

$$\Pi_S = \Pi_{S1} + \Pi_{S2} + \Pi_{S3} + \Pi_{S4} + \Pi_{S5} \quad \textbf{(36)}$$

The energy quantities we are interested in can be calculated on the basis of the entropy quantities and the temperatures. The results of the model are the same as those already presented in Fig.2 (Section II).

IV Conclusion

Applications of thermodynamics in solar energy engineering call for new tools for analysis and optimal design. We must be able to compute processes rather than states. Therefore, we have been looking for a form of introductory thermodynamics which leads to a straight forward approach to the entropy principle.[6]

Modern classical thermodynamics in the form of continuum thermodynamics can be used as the foundation of a theory of the dynamics of heat. We have developed didactic tools which make it easy and natural to use entropy as the fundamental thermal quantity from the start, even in high school and in introductory college physics.[7-9]

Classical thermodynamics is the poor cousin in today's introductory physics courses. Considering that engineers need new tools for new challenges, and that students of ecological economics start wondering about what thermodynamics can teach us about the limits of economic processes,[19] creating a theory of the dynamics of heat may very well help physics instruction to some much needed appeal for a wider audience.

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[12] The analogy is almost perfect. If we stand our usual approach on its head and take “electricity” in the sense of energy instead of charge—as we would if we took the classical form of thermal physics as a guide to theory building—a truly surrealistic version of a theory of electricity is created. H.U. Fuchs, “A surrealistic tale of electricity,” Am. J. Phys. **54**, 907–909 (1986).

[13] S.Carnot: *The Motive Power of Heat*. Translated by R.H. Thurston, Dover Publications, New York 1960. Carnot proposed the image of a fall of heat from high to low temperature, thereby releasing energy—just as water would. Naturally, he did not use words such as entropy and energy. However, his image can be described today using these words.

[14] Eq.13 is the dynamic counterpart of the well known relations which is commonly used to introduce entropy as the “reduced heat” *Q/T*.

[15] H.U. Fuchs, “Entropy in the teaching of introductory thermodynamics,” Am. J. Phys. **55**,215-219 (1987).

[16] The entropy produced is the result of the dissipation of energy which is released as the entropy flows “downhill.” Eq.16 can be used for an alternative derivation of Eq.17.

[17] Reference 6, Chapter 3.

[18] According to this expression, the entropy production rate could turn out to be negative. The problem is solved by considering the emission of radiation by the absorbing body as well as absorption. In our model, emission is included in the heat loss of the collector.

[19] C.J. Cleveland and M. Ruth “When, Where, and By How Much Do Biophysical Limits Constrain the Economic Process?” Ecological Economics **22**,203-223 (1997).