The Missing Link: Introductory Continuum Physics for Engineering Students

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ABSTRACT: Physics instruction faces not only the problem of how to include quantum physics in a curriculum for engineering students, but also of how to deal with increasingly sophisticated usage of classical physics in engineering. Introductory college physics leaves students unprepared when it comes to issues dealing with continuum physics and finite element modeling. Based on a generalized understanding of physical processes—the Continuum Physics Paradigm—the Physics of Homogeneous Dynamical Systems (PHDS) has been built in recent years at Technikum Winterthur. Methods of dealing with laws of balance and a generalized modeling methodology based on system dynamics introduced in PHDS now allow for making the first easy steps toward continuum physics. Students can explore continuum processes in simple system dynamics models which they produce themselves.

I Introduction: Challenges and Foundations

In this paper we investigate steps toward the teaching of continuum physics. Introductory college physics usually ends without touching much on the classical subjects with which serious engineering applications begin. Indeed, the Standard Model of introductory physics instruction is less than useful as a foundation of continuum problems. Therefore, if we wish to move into new territory, we also have to consider reformulating the common introductory courses. Only if we manage to provide a basis for continuum physics from the start will we have a chance to do more in the time allocated to physics in an engineering curriculum.

Considering that continuum physics is a mathematically advanced theory, and that physics instruction faces other problems as well, we may wonder why we should bother at all. However, we will argue that a new unified approach to physics teaching based on what we learn from continuum physics can serve as a basis not only of the more advanced subjects of classical physics, but also of other modern fields such as dynamical systems and quantum physics.[1] Investigating the teaching of continuum physics in college physics therefore opens our eyes to new possibilities for the standard introductory course.

A. Challenges in Physics Instruction

In recent years, chiefly two challenges to physics instruction have been identified and discussed in depth. One is how to deal with so-called misconceptions,[2] the other has to do with the inclusion of quantum physics[3] in the standard curriculum.

However, we believe that physics faces other serious challenges as well. Most important among these is the question of value given to physics in a fast changing world. Put differently, we should start to worry about how to integrate knowledge of the physical world more strongly with the challenges facing human society. The relative importance of physics is decreasing, simply because fields such as biology and social sciences are moving ahead, but also because of an inability of physics and physicists to come up with a new integrative view of nature (and society). Physics is becoming too particular in many respects for a society faced with serious problems.

Seeing that physics does not even serve engineering all that well only underlines what we have been saying. In classical physics, engineers and mathematicians have led the way, and have forged ahead with strengthening the foundations of their applications. Modern continuum physics appears to be all but divorced from the enterprise of physics.[4] Most important in this respect is the development of continuum thermodynamics in the last thirty years,[5,6] which physicists scarcely took notice of. This is all the more distressing since continuum physics not only bridges a widening gap between the sciences and engineering, but can serve also as the foundation of a new understanding of nature.

B. The Continuum Physics Paradigm (CPP)

The basic idea behind continuum physics is simple: physical processes are the result of the flow, the production, and the storage of certain fundamental quantities such as momentum, entropy, charge, or amount of substance. This is the image hidden behind the advanced mathematical treatment given to continuum problems in the literature. The image can be transformed into qualitative and quantitative models of dynamical physical processes accessible to students, not just experienced researchers.

Basically, models of natural and technical processes are produced in a sequence of well-structured steps. The sequence begins with word models describing the flow, production, and storage of the proper quantities responsible for the phenomena. It continues with the construction of system dynamics diagrams representing our ideas, and finishes with a complete mathematical formulation of the relations first expressed in words and graphs.

Chiefly, the CPP teaches us to look for the quantities behind processes for which laws of balance can be expressed. Once these laws have been formulated (in a graphical manner), the next steps consist of constructing the constitutive laws determining the flows and rates of production in the laws of balance.[1]

This approach applies to all fields of physics—from electricity, to mechanics, to thermal physics—which is the source of the integration of subjects which normally are presented as separate entities. As such it considerably revises the standard course on introductory physics. It presents an image of how nature operates which is not so much dependent on the model of the motion of little particles, but rather on the unifying theme of flow and storage in a (spatially) continuous world. The flow of water and air at the surface of the Earth, and of electricity (charge) and heat (entropy), create the image upon which our models of physical processes are built. Mechanics no longer serves as the role model of a theory of physics.[7]

C. The Physics of Homogeneous Dynamical Systems

The first models which we create along the lines of the CPP are those of spatially uniform (homogenous) systems.[1] In the end, they usually make up about 70% to 80% of the content matter of an introductory physics course, with the rest devoted to the physics of electromagnetic fields.[8] It is important to see that homogenous processes—presented on the basis of the CPP—are the perfect foundation for continuum models.

Models of homogenous (dynamical) systems are mathematically simple, and their construction is supported by modern system dynamics tools. They are based on the same fundamental image underlying the more general continuum description of nature. We express them in terms of laws of balance of basic quantities (momentum, entropy, charge, and so forth), and we use constitutive laws for the flows and production rates. Thus, what we learn in the introductory subjects can be transferred easily and completely to continuum models.

D. System Dynamics Modeling in Physics Instruction

The methodology of system dynamics modeling[9]—including the tools created to support it[10]—provides a strong basis upon which the practice of the CPP in introductory physics (high school or college) can blossom.[11] The importance of basing physics instruction on a modeling methodology has been expressed by other researcher as well.[12] Moreover, with its roots in cybernetics and control engineering,[13] a modeling approach based on system dynamics creates a strong link to engineering sciences.

System dynamics tools use the model of spatially homogenous systems as we know them from electric circuit analysis. In their simplest forms, they provide elements representing stored quantities (stocks) and currents or production rates (flows). Together, stocks and flows create the graphical and mathematical representation of laws of balance. We only need another symbol for general variables, and an information connector, to create the (feedback) structures representing the constitutive laws of a model (Fig.1).

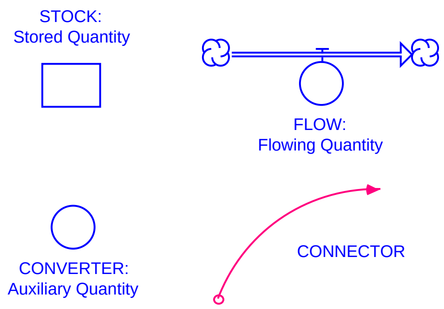

Figure 1: The elements from which system dynamics models of (physical) processes can be constructed. The particular form of the elements presented here is the one used by the program Stella.[10]

Since the elements provided by system dynamics modeling tools are independent of particular fields of applications, we may expect system dynamics to be applicable to just about any field where dynamical processes play a role. Thus, beyond providing a unified basis upon which to build physics instruction, they allow for bringing together such diverse fields as physics, biology, economics and social sciences, and ecology.

E. Steps Toward Continuum Physics Instruction

It is the combination of the Continuum Physics Paradigm and the system dynamics modeling methodology which provides means for entering the world of continuum processes at an early stage, and in a simple manner. Students can learn a great deal about continuum physics if we lead them onto the path of constructing system dynamics models of physical processes in a unified setting. Simply by multiplying and combining models of processes of homogenous systems, important aspects of the nature of extended systems can be captured, modeled, and simulated. A modest amount of formal development can be added to such a course after students have learned about the utility of the modeling approach. Indeed, by investigating the equations which are assembled in the modeling sequence, we may be motivated for the partial differential equation representation of continuous processes. Also, difference equations and finite element approaches can be discussed based on the network models created here.

II A System Dynamics Modeling Approach to Continuous Processes

Processes are the result of the flow, the creation, and the storage of the substancelike quantities such as momentum, entropy, charge, and amount of substance. This simple image can be transferred to useful models with the help of system dynamics modeling tools. Having set up the model of a (continuous) process allows for simulation and investigation of the behavior of the system.

Rather than talking about the philosophy behind the approach, we will present three concrete examples of system dynamics models of continuous processes. (The Stella versions of the models can be obtained from the author.) First, we choose the flow of entropy as an example of a field which is rather simple yet practically important. Second, we take a look at the propagation of sound in air, and finally, we present a model of the flow of charge through a two-dimensional conductor. The discussion of each example should teach us about possible approaches to continuum physics instruction in engineering education, and also make us aware of limitations of the system dynamics tools which may be used.

A. Non-steady-state conduction of entropy in a single spatial direction

The flow of heat through a long bar is the result of the (one-dimensional) conduction of entropy. Since the conduction of entropy is irreversible, it is accompanied by a production rate for this quantity. We can model the entire process by first looking at a segment, or an element, of the long bar, of length Δx. Modeling this section as a uniform body, we can express the fundamental law of balance of entropy[6]

$$\dot{S}_i = \sum_{k=i-1}^{i} I_{s,k} + \Pi_{s,i} \qquad \textbf{(1)}$$

for element i. Here, I_s stands for entropy fluxes, while Π_s denotes the production rate of entropy. Using a system dynamics modeling tool such as Stella, we do not even have to be this formal. The law of balance is expressed by simply drawing the combination of a stock with three connected flows (Fig.2), two of which represent the inflow and the outflow of entropy with respect to the element, whereas the third stand for the rate of production of entropy. The mathematical representation of the law of balance is created automatically after it has been drawn on the screen.

Laws of balance serve as the entry point to a description of dynamical processes. However, they constitute only a small part of the effort we usually expend on constructing a complete model. It is commonly more time consuming to express the constitutive laws for the flows and production rates. Here, the flows of entropy depend upon the difference of the temperatures of adjacent elements, and upon the entropy conductance (which itself may be a function of temperature). The temperatures of the elements are found in terms of the entropy S and the entropy capacity K. The rate of production of entropy for the element is a function of the flow of entropy and the temperatures involved.

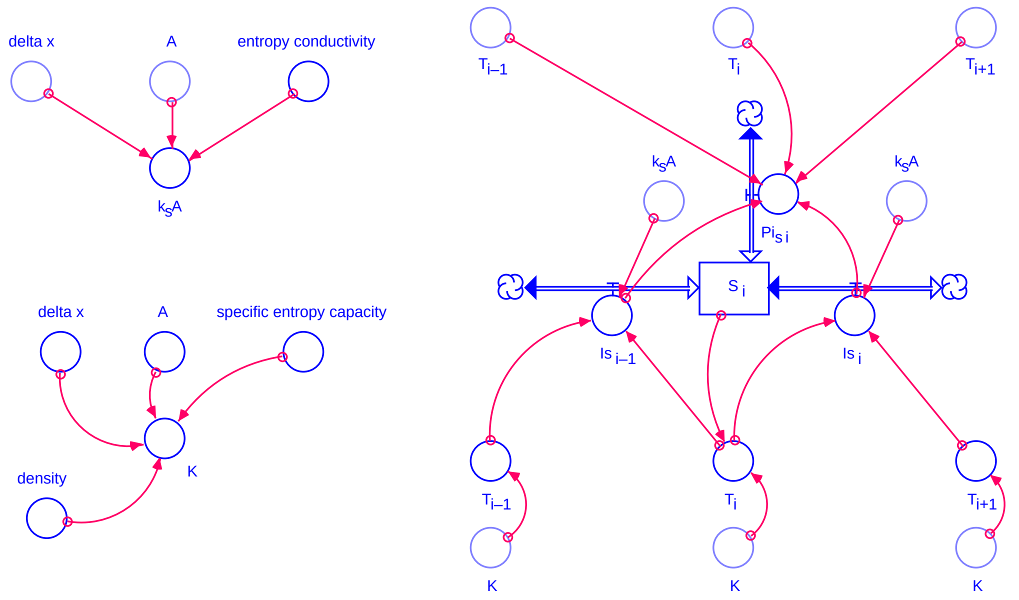

Figure 2: The law of balance of entropy—in Stella notation—for an element of the conducting bar is represented in terms of a stock and some flows. The flows are computed with the help of constitutive laws. On the left side of the model diagram, two important constitutive quantities—the entropy conductance and the entropy capacitance—are calculated.

The model for the extended body is found by connecting submodels such as in Fig.2, one for each element of the bar. We choose a number *n* of elements, reproduce the submodel *n* times, and connect them to form the almost complete model. Naturally, we have to include boundary conditions as well. Just as in the modeling of spatially homogenous parts, where we see that initial conditions for the quantities stored are needed, the graphical representation of the model teaches us about the need for boundary conditions (Fig.3).

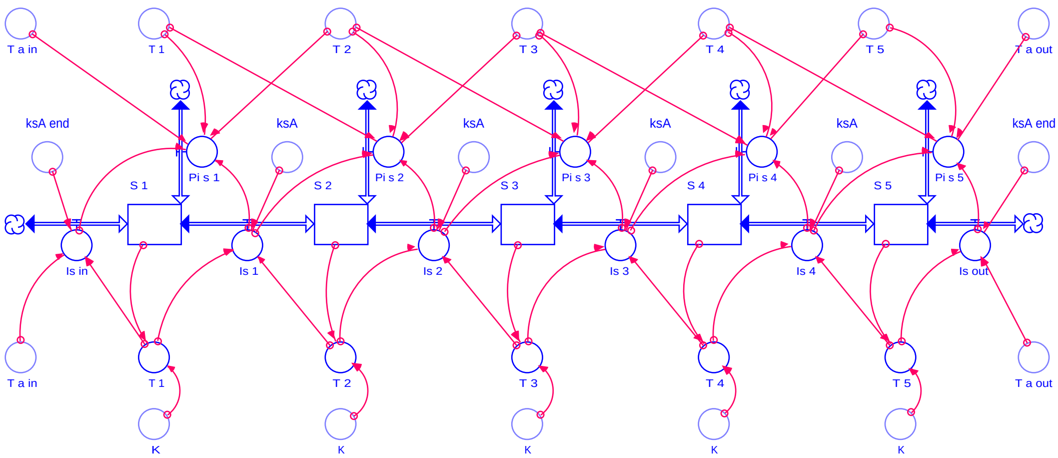

Figure 3: The system dynamics model of the conduction of entropy in a long bar, expressed with submodels for five elements of the bar. Note the boundary conditions at either end of the bar.

Once all the relations have been implemented graphically, and expressed mathematically, the model is complete and can be simulated. Here, we could have the temperatures of the elements presented as functions of time, either graphically or in form of tables.

B. Wave propagation in a one-dimensional elastic medium

Consider air in a long pipe. The propagation of sound in air is the result of the flow of momentum and the production or destruction of volume of the fluid. In addition to these processes, we may have to deal with the flow of entropy; certainly, we will have to use the balance of entropy, even if the processes are considered to be adiabatic.

A system dynamics model of the processes for an element of air in the pipe starts with the symbols expressing the balances of momentum, volume, and entropy. The momentum fluxes from element to element are proportional to the pressure of the fluid at the proper locations, and the rate of production of volume depends upon the relative speed of adjacent sections of air. The rate of change of entropy and of volume of an element determine the rate of change of its temperature according to the fundamental constitutive law for the (ideal) gas:[6]

$$\dot{S} = \Lambda_V \dot{V} + K_V \dot{T} \qquad \textbf{(2)}$$

The constitutive quantities are the latent entropy Λ_V and the entropy capacity at constant volume K_V, which depend upon volume and temperature, respectively.

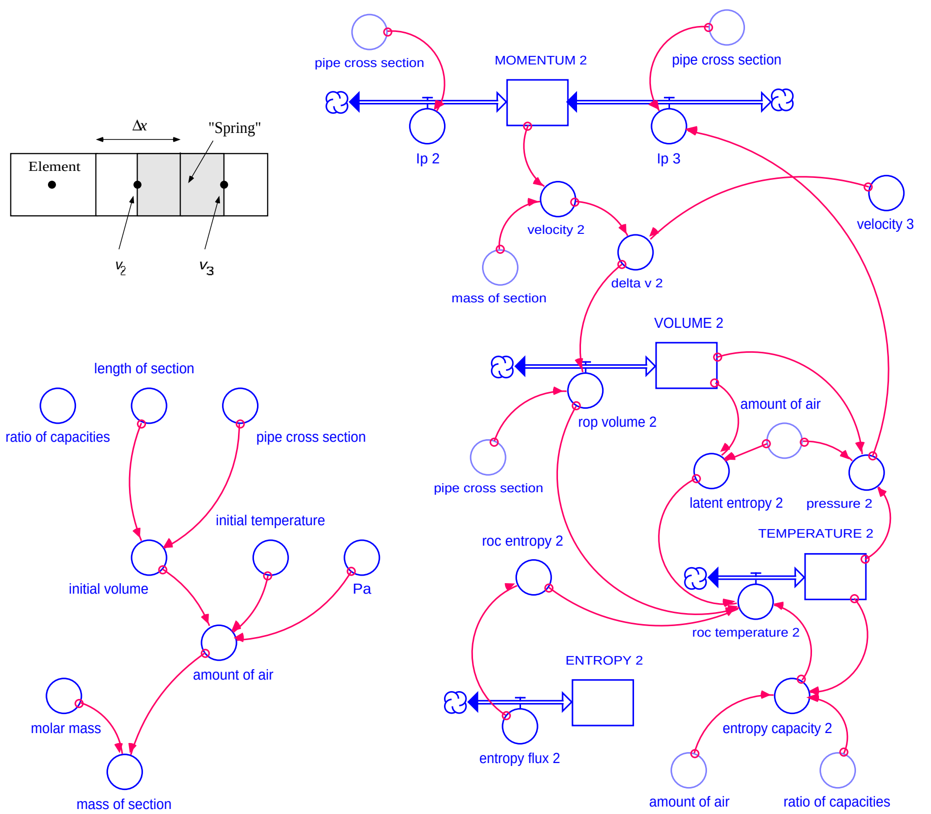

Figure 4: System dynamics model of adiabatic propagation of sound in air, for a single element of air in a long pipe. Note the expressions for the laws of balance of momentum, volume, and entropy. Entropy and volume together determine the rate of change of the temperature, which has to be integrated to yield the temperature. The law of balance of momentum applies to an element, whereas the other relations are expressed for a body of air made up of parts belonging to two elements; these parts are considered to be an elastic "spring" between the elements.

The law of balance of entropy takes a particularly simple form for adiabatic processes: the fluxes of entropy are zero. Rather than including the law of balance expressly, we could have simply set $dS/dt = 0$. However, having the law present in the model makes us aware of the importance of entropy for the propagation of sound, and it shows how the model can be extended to include heat flow.

As in the case of the conduction of entropy, the model of the continuous process is assembled from the submodel for an element of air (Fig.5).

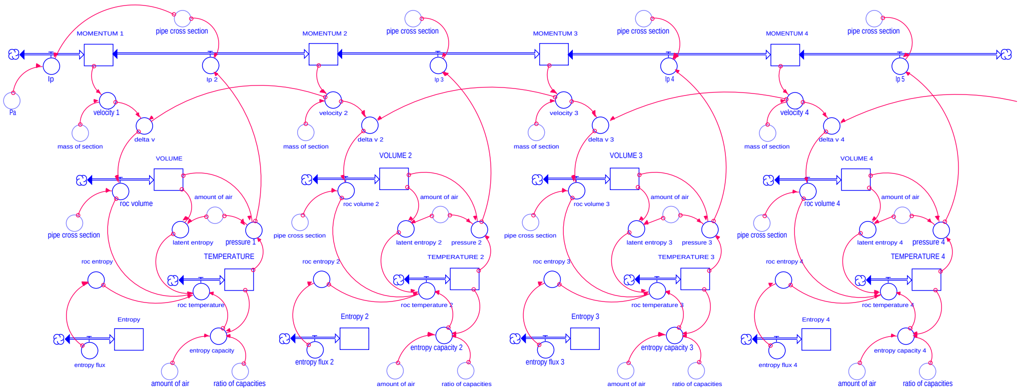

Figure 5: A section of the model of the adiabatic propagation of sound in air in a single direction of space. It is assembled from the model for an element of air shown in Fig.4.

Simulation results of the model can be used to determine the speed of sound in air, and the influence of such things as different boundary conditions and different shapes of pulses travelling through the air can be investigated.

C. The flow of charge through a two-dimensional conductor

Except for the fact that charge is conserved, the conduction of this quantity through a conductor resembles the phenomenon of the conduction of entropy (Fig.3).

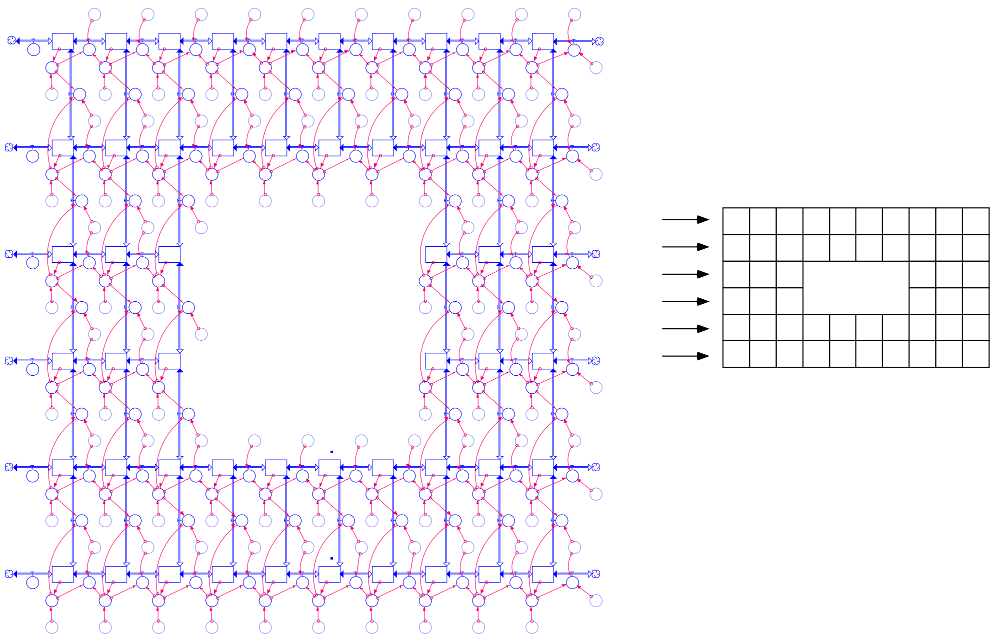

Figure 6: System dynamics model of the flow of charge through a two dimensional conductor. The conductor is rectangular, with a hole at the center. The metal sheet has been divided into square elements, 10 in the *x*-direction, 6 in the *y*-direction. For each element, the law of balance of electric charge is expressed, and the currents of charge are calculated in terms of the differences of potentials. Charge flows into the sheet from the left side, and out of the right.

This model demonstrates the limitations of simple system dynamics tools such as Stella, if we use them to construct finite element models of continuous processes. First, we can easily loose the overview if we move to two dimensions; three dimensions would be virtually impossible to deal with. Second, construction of the model becomes tedious. Third, because models increase in size very quickly, the numerical models become inadequate, and the graphical display of functions is not geared toward the presenting the spatial distribution of the solution. Fourth, and this is rather important conceptually, tools such as Stella are made for dynamical processes, and not so much for steady-state conditions. We would normally treat the flow of charge through a conductor as a purely steady-state phenomenon. Still, it is interesting to see that a system dynamics tool can be used to calculate the spatial steady-state distribution as the solution of a dynamical model for times approaching infinity. We can introduce a virtual charge capacitance of the metal sheet, treat the process as a dynamical one, and wait for the simulation to evolve to the steady state.

III Simple Formal Developments

After having dealt with models of spatially continuous processes practically, students may profit from taking a few steps in the world of formal representations of continuum physics phenomena. We show here how an investigation of the structure of system dynamics models—which are based on models for spatially homogeneous elements of extended systems—can lead to an appreciation of the underlying partial differential equations, including their numerical representations as finite difference or finite element equations (Section IV).

A. The transition from homogenous bodies to continuous systems

The transition from spatially homogenous to spatially continuous descriptions can be made by investigating the role of laws of balance. Take a look at the representation of the law of balance of entropy for a single element as in Fig.2. A tool such as Stella presents the underlying equation in the form

$$S_{i,n+1} = S_{i,n} + \left(I_{s,i-1} - I_{s,i} + \Pi_{s,i}\right)\Delta t \qquad \textbf{(3)}$$

We can transform this difference equation while at the same time introducing the density of entropy of the element of length Δx and cross section A:

$$\frac{\Delta}{\Delta t}\left(A\Delta x \rho_{s,i}\right) = -\Delta I_{s,i} + \Pi_{s,i}$$

or

$$\frac{\Delta}{\Delta t}\left(\rho_{s,i}\right) = -\frac{\Delta\left(I_{s,i}/A\right)}{\Delta x} + \frac{\Pi_{s,i}}{A\Delta x}$$

The latter form demonstrates that in addition to the density of entropy we have to introduce the current density $j_S = I_S / A$, and the density of the rate of production of entropy $\pi_S = \Pi_S / (A\Delta x)$; if we also introduce derivatives instead of difference quotients, we obtain

$$\frac{\partial \rho_s}{\partial t} + \frac{\partial j_s}{\partial x} = \pi_s \qquad \textbf{(4)}$$

which is the partial differential equation representation of the law of balance of entropy for a single spatial dimension. A slightly more formal derivation[14] starts with the general expression of the law of balance for a body:

$$\dot{S} = I_s + \Pi_s \tag{5}$$

introduces densities, and makes use of the divergence law for transforming the integral over the surface into an integral over the volume:

$$\int_V \dot{\rho}_s dV = -\int_V \frac{\partial j_s}{\partial x} dV + \int_V \pi_s dV$$

Transferring the terms from the right to the left side, and setting the integrand zero, leads to the same general expression for the law of balance of entropy for the continuous case as in Eq.4.

B. Derivation of the heat flow equations

Having made the first step, we now introduce continuum physics equivalents of the constitutive laws for the density of entropy, the current density, and the density of the rate of production:

$$\dot{\rho}_s = \rho\kappa\dot{T} \tag{6}$$

$$j_s = -k_s \frac{\partial T}{\partial x} \tag{7}$$

$$\pi_s = -\frac{1}{T} j_s \frac{\partial T}{\partial x} \tag{8}$$

Eq.6 is the capacitance law (κ is the entropy capacity of the material), Eq.7 is Fourier's law of entropy conduction (k_s is the entropy conductivity), and the last relation determines the rate of production of entropy from the dissipation of energy in conduction.

The constitutive laws are used to transform the law of balance into a field equation for temperature. Plugging the last three equations into Eq.5, we obtain the well known field equation

$$\rho c \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial x^2} \tag{9}$$

where $c = T\kappa$ (the specific heat) and $k = Tk_s$ (the energy conductivity) have been taken to be constant.[14]

IV System Dynamics Modeling and Finite Elements

Modeling spatially continuous processes with the help of system dynamics tools brings up the question of the relation of this approach to finite element methods. Also, inspection of the equations appearing as the result of system dynamics modeling of spatially continuous processes leads to an understanding of some simple numerical methods for the solution of partial differential equations. To finish the presentation, we will briefly list a couple of finite element tools which may be interesting for teaching.

A. Similarities and differences

The examples presented in Section II may make us wonder about the relation between finite elements and the modeling approach used here. Obviously, dynamical spatially continuous problems can be solved using tools which were not constructed for this purpose at all. Moreover, the tools reflect a methodology for solving such problems which seems to be different from the common finite element approach.

The difference—and the underlying similarity—of the two approaches can be explained in a simple diagram. Solving spatially extended dynamical problems corresponds to solving many coupled initial value problems, one for each element (Fig.7). Finite element methods, on the other hand, solve boundary value problems which represent the spatial problem. These boundary value problems—one at each point in time—is evolved step by step from the initial time into the future.

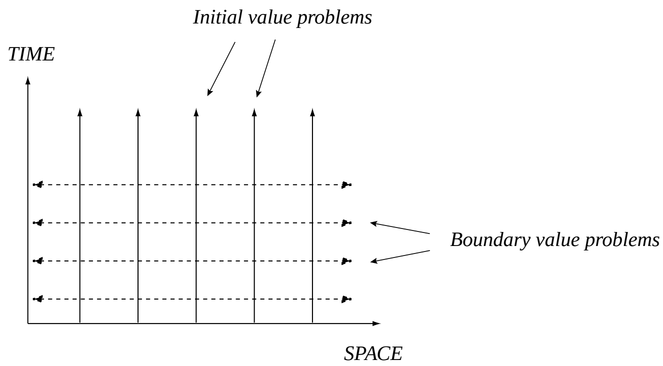

Figure 7: The relation between finite element methods and the method of modeling spatially extended systems using system dynamics tools. System dynamics models calculate initial value problems, i.e. they eveolve many coupled homogenous systems in time. Finite element methods, on the other hand, compute the solutions of boundary value problems which—through time stepping—can also be evolved in time.

A steady-state problem corresponds to a single boundary value problem. This means that such a case cannot be treated directly with system dynamics tools. Rather, we can use them to evolve a dynamical system to a point in time where the solution corresponds to that of the steady-state problem.

Naturally, finite element models and system dynamics approaches must yield the same solutions to the same problem. Indeed, we can read the numerical equations which correspond to the spatial finite difference method for the heat flow equation from the equations as they are assembled by a tool such as Stella, which proves the basic similarity—apart from the form of the numerical methods—of the approaches.

B. Tools for modeling

Students wishing to go further in learning about continuum physics models in general, and about the finite element method in particular, can now use at least a couple of easy to use tools. The first of these is FEHT,[15] a program developed for two dimensional finite element heat transfer (and related) problems. A second one has just been released; it is the Partial Differential Equations Toolbox for MATLAB. The latter program lets us treat general two dimensional problems, including those arising in the mechanics of materials. FEHT in particular is easy to use, but less general and powerful than the MATLAB toolbox. In our view, going all the way into FE modeling may be too much for an introductory physics course, but using these new tools for introducing students to FE methods in engineering may be worthwhile.

V Summary

We have presented a system dynamics modeling approach to spatially continuous dynamical problems as they arise in many continuum physics applications interesting in engineering and the sciences. In contrast to FE modeling, SD modeling derives from the modeling approach developed for spatially homogenous systems. Learning about the latter is common in physics. If we build the teaching of physics upon the Physics of Homogenous Dynamics Systems (PHDS), we create great synergies which can be used to let students take the first steps into the world of continuum physics.

System dynamics modeling is very visual and easy, yet forces the students to go to the foundations of physics and make use of the most general laws. Moreover, it represents an approach which brings out the common core of all the sciences, engineering, and the social sciences, leading to a unified view of dynamical systems wherever we meet them.

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[2] The problem of "misconceptions" has been one of the interesting topics in didactic research for the last two decades. See J.D. Novak ed., *Proceedings of the Second International Seminar on Misconceptions in Science and Mathematic*s (Cornell University, Ithaca, New York, 1987), Vols. I-III. For examples of how physics teaching might be changed to take account of phenomenological primitives, see H.U. Fuchs, "Entropy in the teaching of introductory thermodynamics," Am. J. Phys. **55**,215-219 (1987), H.U. Fuchs, "Thermodynamics - A 'misconceived' theory." In J.D. Novak ed., Vol.III, p.160-167, and H.U. Fuchs, "Do we feel forces?" In J.D. Novak ed., Vol.III, p. 152-159. For a recent overview of the issues of misconceptions and phenomenological primitives see D. Hammer: "More than misconceptions: Multiple perspectives on student knowledge and reasoning, and an appropriate role for educations research," Am. J. Phys. **64**,1316-1325 (1996).

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[6] H.U. Fuchs, *The Dynamics of Heat* (Springer-Verlag, New York, 1996).

[7] Actually, mechanics still serves as a role model, but in a much generalized way. Understood from the point of view of the CPP, Newtonian mechanics was the first example of a successful modern physical theory. We therefore can use the world of motion as a rich source for examples for the CPP approach to physics instruction. However, we need not begin our exposition of physics with mechanics, nor do we explain other phenomena in terms of motion. Rather, mechanics is one of the fields which teaches us about the general structure of models of the physical world constructed in the CPP.

[8] Certainly, electromagnetic field theory is an important ingredient of (classical) physics, and it could serve as an entry into the world of spatially extended systems. Unfortunately, how it—and all that comes before in introductory phyiscs—is presented only serves to underline that we look at physics as a loose assemly of separate fields and theories.

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[10] The best known system dynamics tools are: Stella (High Performance Systems, Inc., Hanover, NH; http://www.hps-inc.com), Dynamo (Pugh-Roberts Associates, Cambridge MA), Powersim (Powersim AS, Isdalsto, Norway; http://www.powersim.no), and Vensim (Ventana Systems, Inc., Belmont MA; http://www.std.com/vensim). A modern tool used in engineering which has some similarity with system dynamics programs is Simulink (The Mathworks, Inc., Natick, MA; http://www.mathworks.com).

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