

EXERCISES

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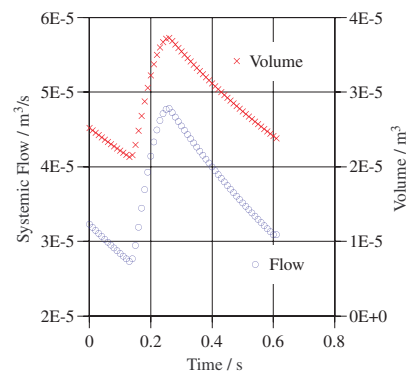
PROBLEMS

1. The flow of blood out of the left ventricle

Consider a section of the aorta of a sheep adjacent to the heart. The amount of blood in that section as a function of time, and the flow of blood out of that section are known (see file PSS_01_P1.xls).

TASKS

- What is the rate of change of blood volume in the aorta?
- What is the flow out of the heart (through the aortic valve) as a function of time during a cardiac cycle?
- How much blood has been pumped out of the heart?



2. Pressure and pressure differences along the systemic circuit

Consider a moment when the pressure of the blood in the left ventricle is 110 mmHg. The pressure difference along the aorta is 40 mmHg (in the direction of blood flow), and the pressure at the end of the aorta is 40 mmHg.

TASKS

- Sketch the pressure of the blood for this moment as a function of position, from the entrance into the left ventricle, into the left ventricle, through the aortic valve, through the aorta, and through the vessels back to the heart (before it enters the right ventricle).
- What is the pressure of the blood right outside of the aortic valve in the aorta?

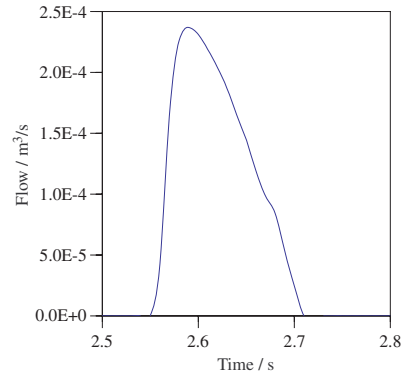
- c. What is the pressure difference along the vessels in the body (from the end of the aorta, to the right ventricle)?

3. Flow of blood from heart to aorta

Assume that the flow of blood through the aortic valve satisfies the relation for turbulent flow.

TASKS

- Sketch the flow characteristic for turbulent flow.
- Formulate the model of turbulent flow as an equation.
- What happens to the current of blood through the valve when the pressure difference across the valve is doubled?
- In the figure, the flow through the valve as a function of time is given. What is the pressure difference as a function of time? (At maximum flow, $\Delta p = 32 \text{ mmHg}$.)



4. Cranial elastance

The table provides measurements of the cranial hydraulic capacitance of sheep as a function of the cranial pressure. (Cranium is the word for the skull of vertebrae, or for the part that contains the brain.)

TASKS

- Determine pressure and capacitance in SI units, make a plot and determine a best fit to the data (use a power law).
- Determine the elastance of the cranium that corresponds to the capacitance relation determined before.
- Use the fit to determine the volume as a function of pressure (use an arbitrary starting value). Make a pressure-volume plot for this cranium.

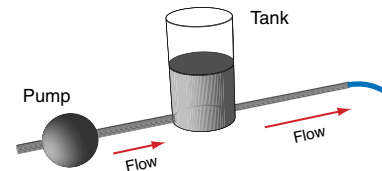
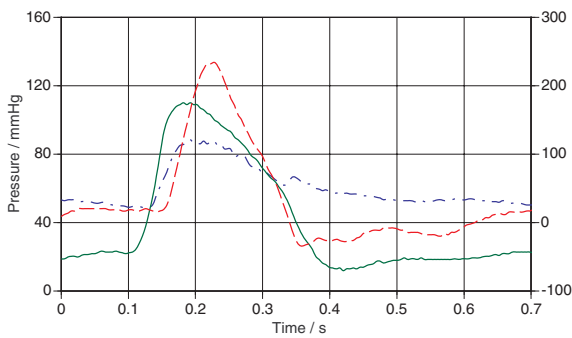
Table 1: Cranial data

p / mmHg	$C_V / \text{ml/mmHg}$	p / mmHg	$C_V / \text{ml/mmHg}$
5.64E+00	1.87E-01	2.32E+01	8.20E-02
6.13E+00	2.35E-01	2.32E+01	6.80E-02
6.63E+00	2.70E-01	3.31E+01	7.23E-02
6.96E+00	2.43E-01	3.46E+01	7.01E-02
9.61E+00	2.02E-01	4.36E+01	6.26E-02
2.06E+01	9.60E-02	4.54E+01	6.58E-02
2.10E+01	1.34E-01	4.99E+01	7.23E-02
2.14E+01	8.42E-02	5.47E+01	6.58E-02
2.22E+01	1.01E-01	5.82E+01	6.26E-02

5. Estimating parameters for the windkessel model of the systemic circuit.

Blood is pumped from the left chamber of the heart (the left ventricle) into the aorta. From there, it flows through the body and back to the heart. In the diagram you find data for the circulatory system of a sheep: blood pressure in the left ventricle (solid line), blood pressure in the aorta (dashes and dots), and volume flow out of the ventricle into the aorta (dashed; measured in the aorta). The cardiac period is 0.61 s.

In a simple model, the left ventricle is represented as a pump, the aorta as a thick, short hose that functions as a storage element, and the vessels going through the body as a pipe having a relatively high fluid resistance (see the figure).



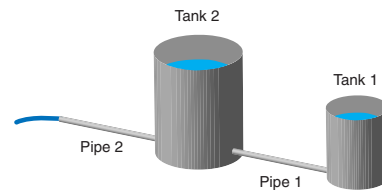
TASKS

- How much blood is pumped by the heart in one cycle? (Take the interval from about 0.16 s to 0.34 s; we neglect the negative values of the blood flow in the diagram.)
- What is the average pressure drop from the aorta through the systemic circuit back to the heart? Note: The pressure of the blood goes to nearly zero at the end of the systemic circuit.
- Estimate the hydraulic resistance that has to be given to this part of the circulatory system in our model. We can assume that the flow is laminar in this part.
- Estimate the value of the elastance of the aorta.

6. Two communicating containers with outflow

Consider two communicating tanks with an additional outflow (see figure and graph below).

The tank is filled with oil having a density of 1000 kg/m^3 . The radius of the larger tank (Tank 2) is 7.5 cm, that of Tank 1 is 5.0 cm. The flow resistance of the pipe leading into the open (Pipe 2 in the figure) has a value of $1.0 \cdot 10^9 \text{ Pa}\cdot\text{s/m}^3$.



TASKS

- Explain why the levels of oil in the tanks are as shown in the graph below. In particular, determine which function in the graph belongs to which tank, and explain why the maximum of the curve starting at a low level occurs where the two functions cross.
- Use the data for the level of oil in the Tank 2 to estimate the time constant of the large tank with its two pipes.
- Use the time constant (and other system parameters) to determine the value of the flow resistance of the pipe between the two tanks.
- Determine the rate of change of volume for the two tanks (separately) for $t = 500$ s. Use these values to find the flows of oil through the two pipes at the same moment.

