Chapter 1

Storage and Flow of Fluids

1.1 Processes

1. Why does the phenomenon shown in Fig. 1.1 demonstrate that levels of liquids rather than quantities of liquids equilibrate in communicating tanks? What does this tell us about the driving force of the flow of fluids?

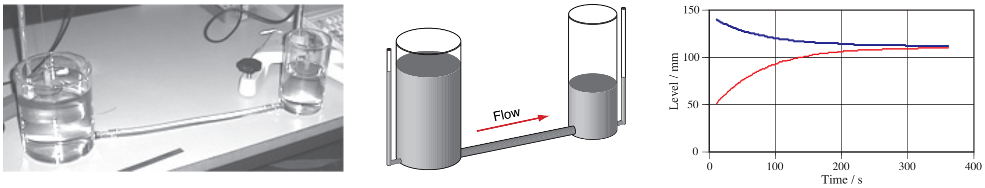


Figure 1.1: Two tanks containing rape seed oil are connected by a hose at the bottom, the liquid flows from the one having the higher fluid level to the one having a lower level (this is independent of the size of the tanks; levels equilibrate, not quantities of liquid). Right: Data for the levels of rape seed oil.

2. Describe and explain the phenomenon shown in Fig. 1.1, i.e., create a word model of the system and the processes it undergoes. What concepts and ideas are used in the description and explanation?

3. If the oil level is higher in the thin tank of Fig. 1.1 at the beginning, and if there is less oil in the thin tank compared to the wide one, will the oil flow from the thin to the wide tank or the other way?

4. What is the ratio of the cross sections of the two tanks used in the experiment reported in Fig. 1.1?

5. What happens to the data shown in the graph of Fig. 1.1 if you make the pipe between the tanks longer? If you give the pipe a larger diameter? Sketch your expectation in a diagram similar to the one in Fig. 1.1.

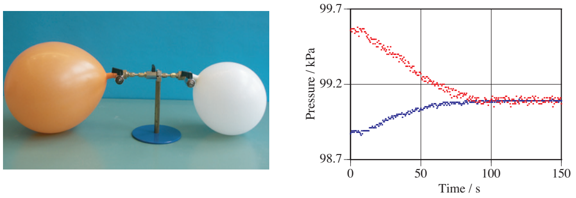
6. Why does the photograph of the U-pipe with water and oil (Fig. 1.2) demonstrate that fluid pressures rather than levels of liquids equilibrate in communicating tanks?

7. If you take the phenomenon of equilibration in balloons as in Fig. 1.3, what

Figure 1.2: With oil on top of water in the right column of the U-pipe, the levels of the liquids are not the same after equilibration. The effect is observed as well if the two tanks have different cross sections and/or different shapes.

does this demonstrate about the driving force for the flow of air? Note that the balloons are at the same level, air flows horizontally.

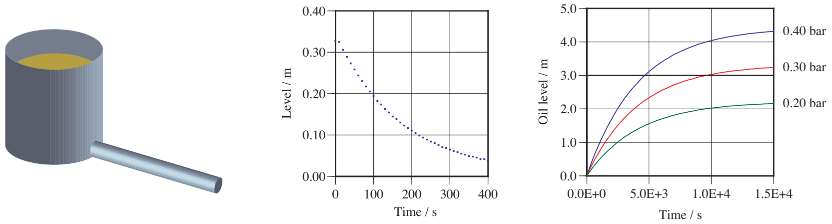
Figure 1.3: When air is allowed to flow freely between two communicating balloons, neither volumes nor levels are the same after equilibration. As the diagram on the right shows, the pressure of the air in the balloons equilibrates. The upper curve in the diagram belongs to the smaller balloon (surprisingly!).



8. Describe the process of draining of a tank such as the one in Fig. 1.4. Produce a word model for the process.

9. Consider the diagram of discharging of an oil tank shown in the middle of Fig. 1.4. What would be the curve showing discharging of a tank that has twice the cross section? What would the curve be for a pipe having half the length?

Figure 1.4: A straight-walled tank filled with oil is discharged through a horizontal pipe at the bottom (center diagram) or charged with the help of a pump (diagram on the right). The measured levels closely fit exponential functions of time.



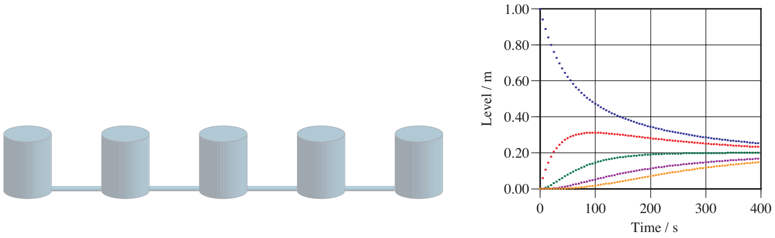
10. Assume the tank used in the process shown in Fig. 1.4 to have a cross section of 0.50 m^2. Convert the diagram (in the middle) to one showing the volume of liquid as a function of time.

11. In the diagram in the middle of Fig. 1.4, measure the time it takes for the fluid level to fall to half its initial value. How long does it take from there for the level to fall to 1/4 its initial value?

12. Create a word model for the process of filling of an oil tank as in Fig. 1.4 (diagrams on the right). What is the difference of the three situations shown? How can the curves be compared to those observed for draining?

13. Why don't the levels in the tanks on the right in Fig. 1.5 change right from the beginning?

Figure 1.5: A chain of tanks filled with water. Water stands at different levels at first (the four tanks on the right are virtually empty at first). When the pipes are opened, water levels change with time.



1.2 FLOWS, RATES OF CHANGE, AND BALANCES

14. Determine the mass of 1 m^3 of water. What is its amount of substance of this quantity of water?

15. Sketch a current that is constant at first, and then decreases at a constant rate.

16. Describe the current shown in the diagram on the left in Fig. 1.6. Do the same for the diagram on the right.

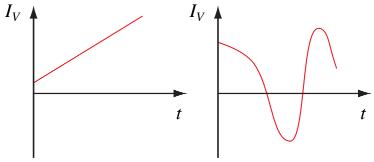


Figure 1.6: Volume currents as a functions of time. Currents can change slowly or quickly, they can be positive or negative.

17. Oil flows through a pipe with a constant volume flow rate (volume current) of 0.0020 m^3/s. The cross section of the pipe widens from 20 cm^2 to 40 cm^2. The density of the oil is 900 kg/m^3.
    a. What is the value of the mass flow (current of mass)?
    b. What do you need to know in order to determine the current of amount of substance?
    c. What are the values of the volume flows in the narrow and in the wide parts of the pipe?
    d. How much oil flows through a cross section of 30 cm^2 in 200 s?

18. Consider the pipe in Exercise 17 as a system. How strong are the volume currents at the narrow entrance and at the wide exit?

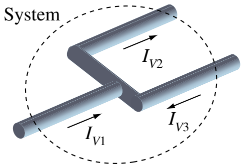


Figure 1.7: A system of pipes having a junction. Two of the flows are known.

19. A current of liquid is constant. Write the equation for the volume transported with this current for a chosen interval of time Δt.

20. A current of water changes linearly from 10 liters/s to 4 liters/s in 20 s. How much water has been transported during these 20 s?

21. The volume flow in Exercise 17 increases steadily within the first 100 s from 0.0020 m^3/s to 4.0 l/s and then remains constant. How much oil flows through the pipe in the first 120 s?

22. The volume of water in a tank increases quickly from 10 liters (Point 1) to 20 liters (Point 2). Then it decreases slowly to 15 liters (Point 3). What is the change of volume from Point 1 to Point 2? From 2 to 3? From 1 to 3?

23. The volume of water in a tank changes linearly from 20 liters to 12 liters in 10 s. What is the rate of change of volume at time 5 s? What is the average rate of change for the entire period?

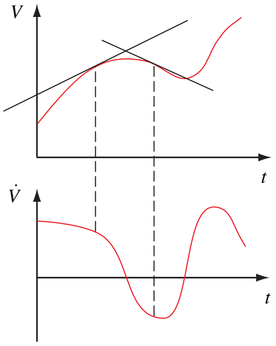


Figure 1.8: Determining the rate of change of volume by calculating the slope of tangents to the *V*(*t*) curve. The symbol *V* with the dot above it denotes the rate of change of *V*.

24. Are there points where the rate of change of volume is equal to zero for the volume sketched in Fig. 1.8? If so, what is the property of those points?

25. The rate of change of volume of a liquid in a container equals – 2.5 m^3/s. By how much does the volume change in 2 minutes?

26. Explain the procedure for calculating the volume of liquid in a tank from its rate of change. What other information is needed to perform this calculation?

27. A tank has neither inlets nor outlets for water. Express both the instantaneous and the integrated forms of the law of balance of volume of water for this tank.

28. There is an inflow of liquid to a tank. At the same time, liquid is flowing out.

The outflow is equal in magnitude to the inflow. What is the law of balance of volume for this situation?

29. A pipe is branching. What is the relation between the three currents related to the junction? How does this junction rule follow from the law of balance of volume?

30. Why is it important to clearly identify the systems or elements for which a law of balance is to be written? Explain what can go wrong if this is not done carefully.

31. During a period, 100 liters of water flowed out of a tank. At the same time, 20 liters flowed in through a pipe. During the same period, the volume of water changed by + 30 liters. Explain how this is possible.

32. Explain the meaning of the interaction rule. If there are three tanks exchanging a liquid, how many times does the interaction rule apply?

33. What happens to the law of interaction formulated in Fig. 1.9 if the two tanks are treated as a single element?

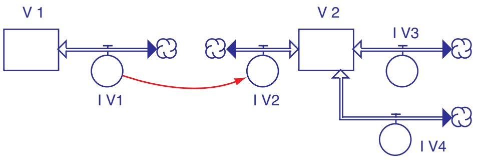


Figure 1.9: Representation of laws of balance in system dynamics diagrams. The thin arrow denotes the interaction rule (action and reaction).

34. The magnitudes of the volume currents I_{V1} and I_{V2} in Fig. 1.7 are 10 l/s and 15 l/s, respectively.
    a. What is I_{V3}? What law is needed to determine this?
    b. Determine the three volume currents at the system boundary.
    c. Within 100 s, the volume current I_{V1} falls steadily from 10 l/s to 4 l/s. I_{V2} stays constant How much oil flows through the third pipe in this time?

35. Water flows through a pipe into a trough and out again through two pipes. The inflow is a constant 5.0 l/s and one of the outflows measures 2.0 l/s.
    a. How much water flows in or out within ten minutes with the two currents whose strengths are known? Is it possible to use these numbers to calculate how much the water volume in the trough changed in this time span?
    b. The rate of change of the volume in the trough is 2.0 l/s. What is the second outflow?
    c. Calculate the change of volume of the trough within ten minutes, first with the help of the rate of change and then using the three amounts transported by the current.

36. The rate of change of the volume of oil in a tank decreases linearly within 60 s from 0.0010 m^3/s to – 0.0020 m^3/s. The tank has both an inlet and an outlet.
    a. Determine the formula for the rate of change of volume as a function of time. Do the same for the net current.

b. The inflow measures a constant 2.0 l/s. Determine the current at the outflow as a function of time.

c. Find the amount of oil exchanged at the outlet in 60 s.

d. Initially, there is 100 l of oil in a tank. What are the contents of the tank as a function of time?

1.3 PRESSURE AND PRESSURE DIFFERENCES

37. What does the diagram on the left in Fig. 1.10 tell us about the pressure of a liquid as a function of depth?

38. Determine the ratios of the slopes of the straight lines in the diagrams on the right in Fig. 1.10. Do the ratios agree with the ratios of the densities of water, olive oil, glycerine, and alcohol? What would the slope be for mercury?

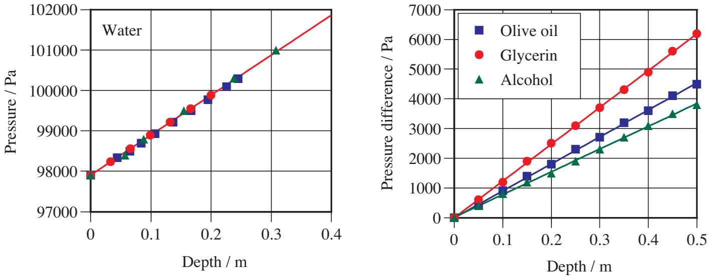


Figure 1.10: Pressure versus depth in water in tanks of different cross sections (diagram at center). The slope of the linear fit is close to 10,000 SI units. The pressure difference measured from the surface as a function of depth in different liquids (right). The linear fits have different slopes (higher density yields steeper slope).

39. Why is the pressure-altitude relation for the air in our atmosphere (Fig. 1.11) not linear? What does the slope of this relation at the beginning tell us about the density of air at sea level?

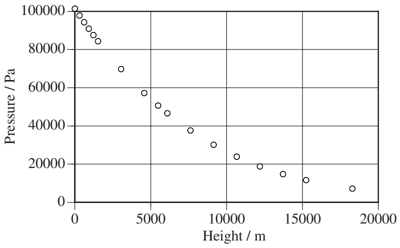


Figure 1.11: Pressure as a function of height in the atmosphere. Values correspond to the so-called standard atmosphere. In reality, conditions change with time and place.

40. There is water in a U pipe which is open on both ends. A certain amount of a liquid with an unknown density is added to the water in one leg of the U pipe (Fig. 1.2). It floats on the water without mixing with it. All the levels as well as the level of the dividing line between the two liquids are measured. How is the density of the unknown liquid determined?

41. Oil having a density of 800 kg/m^3 is stored in a straight walled tank.

a. What is the pressure gradient in the vertical direction?

b. What is the pressure gradient if the tank is spherical in shape?

42. What is the pressure difference of water at rest inside a very long pipe (10 km) between the two ends. One of the ends is 10 m higher than the other end.

43. The pressure of the air in the atmosphere 5 km up is one-half that at the surface. How much higher do you have to go for the pressure to decrease by another factor of two?

44. What is the meaning of gravitational potential? What is the meaning of differences of the gravitational potential? Compare to pressure differences.

45. Trace the pressure of a water in the following system. Water is taken from a lake at a certain depth. It is pumped through a horizontal pipe on land by a first pump. After a certain distance, a second pump is used, and the water is pumped into a lake higher up on a hill.

46. Oil is pumped around in a hydraulic circuit having a pump, an engine, and some pipes. If the pressure difference across the engine is 2.0 bar, what is the pressure difference across the pump?

47. Take a point in time when the blood leaving the left ventricle of a heart has a pressure of 130 mmHg. The pressure difference across the arteries and smaller vessels through the body is 70 mmHg. Before the right ventricle, the pressure of the blood is 20 mmHg. How is this possible?

48. Oil flows through a pipe leading upward. What processes lead to the pressure difference across the pipe in the direction of flow? What is the sign of the pressure change in the direction of flow?

49. Water flows with a constant volume current through a pipe. The pressure at the inflow is measured at 2.5 bar. At the point of outflow it is 1.2 bar. The pipe is 200 m long.
    a. What is the pressure difference along the pipe?
    b. What is the pressure gradient?
    c. What is the pressure 60 m from the input?

50. A pipe with oil flowing through it branches off at one point. A little later, the two branches come together again. What are the pressure differences along the parallel parts of the pipe compared to each other?

51. What is the algebraic sign for the pressure difference in a pump in the direction of the volume current?

52. A pump draws water out of a shallow, open topped container and pumps it through a wide pipe and then a narrow one before it is released into the air again.
    a. Sketch a diagram of the pressure from the container to the point where the water is finally released.
    b. The pressure difference across the first part of the pipe is – 0.70 bar; along the second part it is $-8.0 \cdot 10^4$ Pa. How much does the pressure in the pump increase?

53. Why can we tell that there must be a pressure difference in the system in Fig. 1.12 between points B and C?

1.4 FLOW RELATIONS

54. What is considered to be the cause of the pressure drop in the long horizontal pipe in Fig. 1.12? What is the pressure difference along the pipe if the fluid does not flow (if the pipe is plugged)?

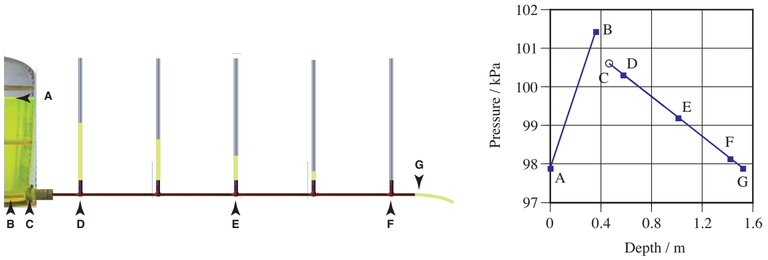


Figure 1.12: Water is flowing out of a large tank through a long, horizontal pipe. This is an example of a hydraulic circuit leading from A through B, C, …, to G, and back through the air to A. Different water levels allow for the pressure of the fluid to be measured along this circuit. Pressure of the liquid in the system at different points along a circuit, plotted as a function of position. Note the drop from B to C.

55. Consider the type of flow characteristic found for oil flowing in a pipe as in Fig. 1.13 (in the left diagram). What do you expect the characteristic curve to be for a pipe twice as long as the one used in the experiment? What do you think the characteristic curve should be if the pipe has a larger radius?

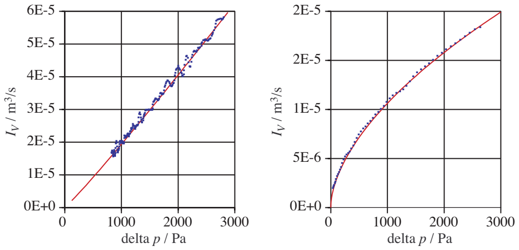


Figure 1.13: Flow characteristics for laminar flow (oil in a pipe, left) and for turbulent flow (water in a pipe, right). I_V is the symbol for volume current.

56. Oil flows through a pipe with a volume current of 0.01 m^3/s. The pressure decreases by 0.50 bar along the pipe. What is the hydraulic resistance of the system of oil and pipe? What is the hydraulic conductance?

57. Olive oil flows through a pipe. Measurements show that the drop in pressure along the pipe is proportional to the volume current.
    a. Sketch the characteristics of this flow (the diagram showing volume current versus pressure difference).
    b. What statement can be made about the hydraulic resistance of the flow in the pipe?

58. For a given situation of oil between two plates (Fig. 1.14), how strongly does one have to pull if one wants to move the upper plate twice as fast? Compare this relation to the flow characteristic of oil through a pipe (Fig. 1.13).

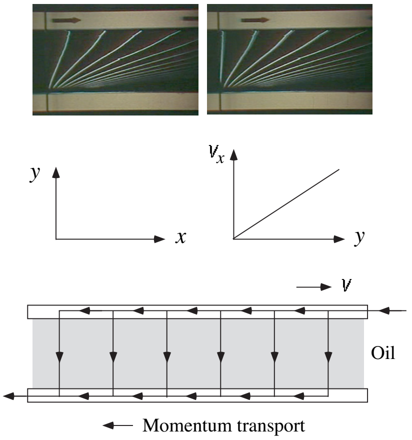


Figure 1.14: Viscosity is the factor relating sheer stress in a fluid (j_p) and the gradient of speed perpendicular to the motion of fluid layers (dv/dy). The photographs at the top show bubbles in oil that is moving as the result of the motion of the upper plate.

59. How could you use oil flow through a horizontal pipe to determine the viscosity of the oil?

60. By how much does a turbulent flow change when you double the pressure difference (Fig. 1.13, right)?

61. The pipes in Fig. 1.7 in Exercise 17 have hydraulic resistances of $50.0 \cdot 10^6$ $Pa \cdot s/m^3$, $20.0 \cdot 10^6$ $Pa \cdot s/m^3$, and $30.0 \cdot 10^6$ $Pa \cdot s/m^3$ when a certain oil flows through them. The pressures at both inlets (for pipe 1 and pipe 3) are equal.

a. What is the total resistance of this arrangement of pipes?

b. The volume current through pipe 1 is 0.0020 m^3/s. Determine the pressure difference along pipe 3.

c. What is the volume current through pipe 3?

d. Find the pressure difference along pipe 2.

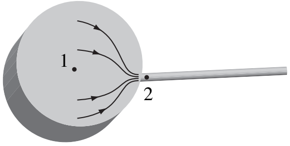


Figure 1.15: A tank and pipe viewed from above. The fluid enters a small cross section from the wide cross section of the tank. The pressure of the fluid drops from point 1 to point 2.

62. Explain why there should be a pressure difference associated with the situation represented in Fig. 1.15. What is the process associated with the pressure change?

63. Is the pressure change in Equ. 1.1 a temporal change or a spatial change?

$$\Delta p_B = -\frac{1}{2}\rho\left(v_2^2 - v_1^2\right) \tag{1.1}$$

64. When blood flows out of the left ventricle of the heart into the aorta, does its pressure change only because of flow resistance?

1.5 PUMPS

65. Sketch a characteristic diagram of an ideal (continuous flow) pump.

66. Investigate the operation of the heart of a human and sketch the pressure-volume diagram of blood in the left of right ventricle of such a heart.

67. What does the process diagram of a pump depict? Compare it to the process diagram of resistive flow of a fluid through a pipe.

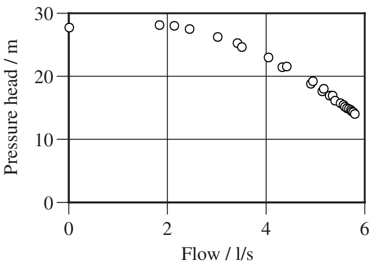


Figure 1.16: Example of a pump characteristic. In the simplest case of constant pressure difference independent of the flow, the characteristic curve would be a horizontal line. Pressure is given as the height through which water can be pumped.

68. Consider the pump characteristic in Fig. 1.16. How high can this device pump water if the flow is to be maintained at 4 liters/s?

69. We wish to pump water some 40 meters high. Consider pumps whose characteristic is shown in Fig. 1.16. What could you do to achieve your aim? How big could the flow of water be made?

70. Water is to be pumped through a horizontal pipe with the help of a pump whose characteristic looks like the one shown in Fig. 1.16. The flow of water through the particular pipe has a characteristic relation like the one seen in Fig. 1.13 (on the right). How can you use the characteristics to graphically determine the magnitude of the current of water established by the pump?

1.6 STORING FLUIDS

71. What does the pressure-volume relation of a cylindrical tank with a liquid stored in it look like? What is the relation for a cylindrical tank with a smaller diameter than that of the first tank?

72. Imagine a spherical tank for liquids. The tank is open to the air. What would be the pressure-volume characteristic diagram of this storage device?

73. Why is the second characteristic relation shown in Fig. 1.17 linear? Why do the first and the last of the containers shown in Fig. 1.17 have capacitive characteristic curves that rise more steeply for bigger volumes of liquids stored?

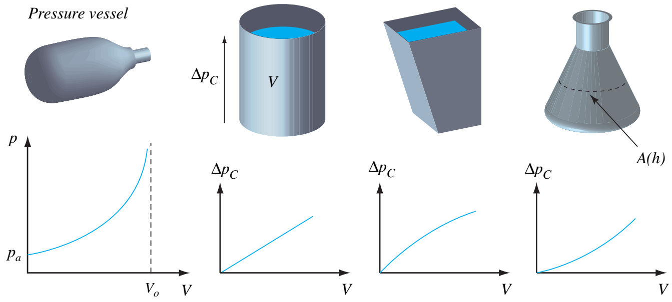


Figure 1.17: Different containers have different capacitive characteristics. The hydraulic capacitance of an open container is proportional to the (variable) cross section.

74. Explain the meaning of elastance and hydraulic capacitance.

75. Sketch the capacitive characteristic relations for two cylindrical tanks, one having a large cross section, the other having a small cross section.

76. Explain how to calculate the change of volume for a container having a nonlinear characteristic relation.

77. The aorta of a human (or of a mammal, for that matter) is an elastic storage device for blood. Sketch a pressure-volume relation for blood in the aorta. How does the relation of an older person compare to that of a younger one?

78. Use the qualitative sketch of the pressure-volume relation of a toy balloon (Fig. 1.18) to explain why the air can flow from a less inflated balloon to a more inflated one. (Remember that we would normally assume the more inflated balloon to have the higher pressure.) Is it possible for the air to go from the more inflated one to the less inflated one? (Fig. 1.3)

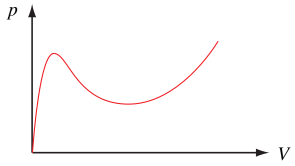


Figure 1.18: Qualitative sketch of the pressure-volume relation of a typical toy rubber balloon.

79. How much water must be added to a straight walled tank so that the pressure at the bottom rises by 0.020 bar? The cross section of the tank is 0.30 m^2. What is the answer if the area of the cross section is twice that? If it is half that?

80. The volume of olive oil in a straight walled tank with a base area of 0.050 m^2 has changed by 500 l. By how much has the pressure of the oil at the bottom of the tank changed?

81. Determine the hydraulic capacitance for the following straight walled containers (using SI units):
    a. A petroleum tank with a diameter of 15 meters.
    b. A vertical pipe filled with mercury with a radius of 0.50 cm.
    c. A U-pipe containing a liquid.

82. At a certain fill level, a container has a hydraulic capacitance of 10^{-5} m^3/Pa. How fast does the pressure at the bottom of the container rise when the volume changes at a rate of 20 l/s?

83. A cone shaped container for storing water has a radius of 10 cm at its bottom. One meter higher, its radius is 40 cm.
    a. Sketch the pressure as a function of the fill level.
    b. Sketch the hydraulic capacitance as a function of the fill level.
    c. Sketch the pressure as a function of capacitance.
    d. With the help of the third diagram (P-C_V), graphically determine the change of volume when the fill level changes from 50 cm to 1.0 m. Use geometrical formulas to determine the same result.

84. What happens to the pressure of a fixed quantity of air if its volume is reduced to half the initial value (at constant temperature)?

85. Describe the everyday experience that tells us that the compressibility of air decreases with increasing pressure.

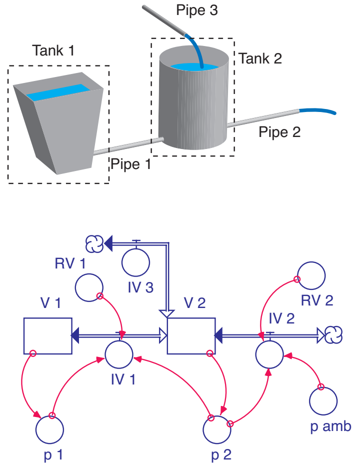


Figure 1.19: A system dynamics model diagram for a system of two fluid tanks (stocks) and several pipes (flows). It is assumed that the fluid is oil and that the Bernoulli effect has been neglected.

1.7 DYNAMICAL MODELS

86. In what sense can we say that a model such as the one in Fig. 1.19 explains how a system functions?

87. Explain the meaning of the structure of stocks (rectangles) and flows (thick arrows) in the system dynamics diagram of Fig. 1.19. Why are there three flows connected to the stock V2? Why are p1 and p2 connected to the quantity labeled IV1?

88. To what percentage of the final level does the level on the right in the diagram of Fig. 1.20 rise in one time constant?

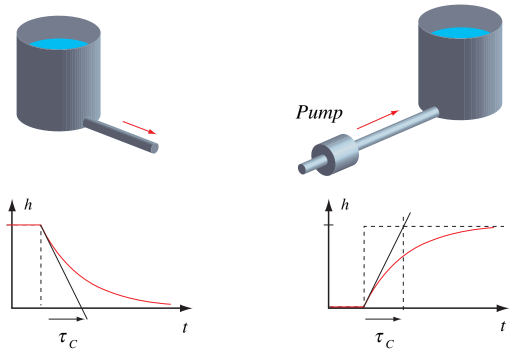


Figure 1.20: Draining or filling of straight walled tanks through pipes showing laminar flow leads to exponentially changing functions. The initial rate of change is used to define the time constant of the system.

89. Consider a system of two communicating tanks with oil as in Fig. 1.1. How can you use the diagram there to graphically determine the time constant of the system? What is your estimate of this time constant?

90. Assume you have observed the level of oil in a tank that is drained (center of Fig. 1.4) but forgot to measure other quantities. Why is this observation not enough to independently determine the capacitance of the tank and the resistance of the pipe?