Chapter 2

Electrical Systems

2.1 Processes

1. Consider the experiment described in Fig. 2.1. Do the electrometers connected to charged spheres measure quantities of electricity (charge) or the intensity of electricity (voltage)? Why?

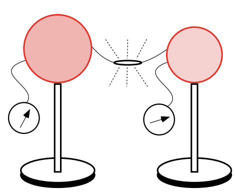

Figure 2.1: Differently charged bodies in (electrical) contact. A glow lamp lights up briefly, and the readings of the electrometers become equal.

2. Why does the phenomenon shown in Fig. 2.2 demonstrate that voltages rather than quantities of electric charge equilibrate in communicating capacitors? What does this tell us about the driving force of the flow of charge?

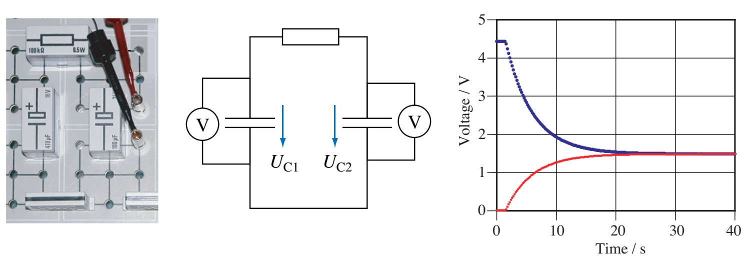

Figure 2.2: Two capacitors are connected with a resistor to form a circuit (circuit diagram is at the center). The diagram on the right shows the voltages of the capacitors as functions of time.

3. When you rub non-conducting materials (plastic, rubber, glass, amber) on fur or cloth, static electricity develops. Where does the charge come from? Has it been produced?

4. A metal coated pingpong ball is hanging from a thin thread. It can be charged with the help of a plastic rod rubbed against fur. After this, when we bring the rod close to the ball (without touching it), the ball is repelled. If we bring a glass rod rubbed with a silk cloth near the charged ball, it is attracted by the rod. How can this be explained?

5. Do generators, power supplies, and batteries generate electricity (i.e., electric charge)? What do they do with charge?

6. Consider the circuit in Fig. 2.3. Should the charge currents through the two lamps be equal? What relation do they satisfy in general?

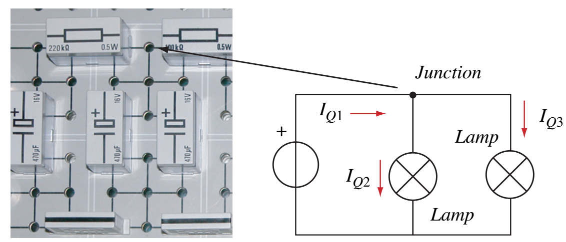

Figure 2.3: A branching circuit with two loops. A charge current flows toward a junction and splits into two which add up to the original current.

7. In Fig. 2.4 (right), the electric potential in a single loop of a circuit is shown as a function of position along the circuit. How many voltages are there? Identify them in the diagram on the right. What is the relation between the voltages?

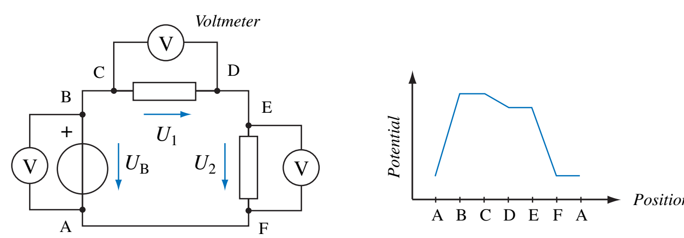

Figure 2.4: Voltages across the elements in a single loop circuit (the elements are connected in series). The voltmeters are connected in parallel to an element. Voltages (potential differences) are denoted by (blue) arrows parallel to the devices. The rectangles in the circuit diagram symbolize resistors.

8. Consider the circuit in Fig. 2.3. What must be the relation between the voltages across the two lamps? Why?

9. Which fluid quantities can be compared to which electric quantities? How are they similar? Are there differences? Are there elements of fluid systems (such as pipes or pumps) that are analogous to elements of electric systems?

10. Sketch an electric circuit that is analogous to the system of two communicating fluid tanks. Do the same for a system of two communicating tanks where one of them has an additional outflow.

2.2 CHARGE, CURRENTS, AND BALANCES

11. A wide piece of tape is torn off an uncharged metal sphere (the sphere is insulated electrically). Afterwards the metal sphere shows an electric charge of $2 \cdot 10^{-10}$ C. How does this occur? Is the tape also charged? If so, how much charge does it have?

12. A neutral oxygen atom loses two electrons. What is its charge then?

13. A charge current of 2.0 A flows through a wire. How much charge flows

through a cross section of the wire in 10 s?

14. The current of charge through a resistor is variable and increases linearly in 100 s from 0.20 A to 1.0 A (as in the diagram on the left of Fig. 2.5). How much charge has flowed in this time through a cross section of the resistor?

15. Describe the current in Fig. 2.5 in the diagram on the right. How do you use the diagram to calculate the charge transported during a period of time?

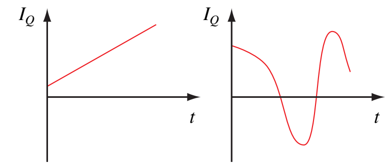

Figure 2.5: Currents of electric charge as a functions of time. Currents can change slowly or quickly, they can be positive or negative (this has nothing to do with positive or negative charge).

16. An alternating current flows through a wire that behaves according to

$$I_q(t) = a\sin(2\pi f t)\ ,\quad a = 2.0\text{A}\ ,\quad f = 50\text{Hz}$$

f is the frequency. How much charge flows in a cross section in a half-period? How much flows in a whole period?

17. The rate of change of charge of a capacitor is a constant – 0.20 C/s. What is the charge of the capacitor at $t = 10$ s if it was equal to 3.0 C at $t = 0$ s?

18. There are two points in the diagram on the left of Fig. 2.6 where the rate of change of charge of a system equals zero. What happens to the charge of the system at those points?

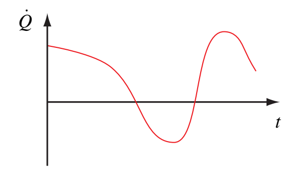

Figure 2.6: Raste of change of charge of an electric system as a function of time.

19. Explain howe the charge of an electric system is determined from a knowledege of the rate of change of charge.

20. The second capacitor in Fig. 2.7 receives a quantity of charge of 2.0 C during a certain period from the first capacitor. During the same period, it loses 3.0 C through the second resistor. What are the changes of charge of the capacitors (separately)?

21. Use an explicit expression for the interaction rule for the capacitors in Fig. 2.7. What is the form of the laws of balance of charge for the capacitors in this case?

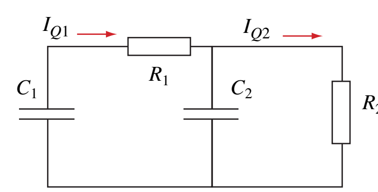

Figure 2.7: Sketch of an electric circuit having two capacitors and two resistive elements.

22. One often introduces a third electric current in the diagram of Fig. 2.7 leading from the junction to the second capacitor. Call this current I_{Q3}. Rewrite the laws of balance of charge for both capacitors. Is the system of equations still equivalent to what was formulated above (Exercise 21)?

23. Sometimes the junction rule is formulated as follows: The sum of all currents meeting at a junction is equal to zero. How does this agree with the example shown in Equ. 2.1 (as it might apply to the junction in Fig. 2.7)?

$$I_{Q1} = I_{Q2} + I_{Q3} \tag{2.1}$$

24. Combine the capacitors in Fig. 2.7 into a single one (for this to make practical sense, one would make the resistance of R1 small compared to that of R2). Rewrite the law(s) of balance for this case.

25. A wire leads from a battery and then branches off into two parts. The current coming from the battery measures 0.50 A. A current of 0.70 A flows through one branch of the wire from the junction. What is the current in the second wire and in what direction does it flow?

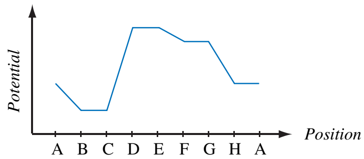

Figure 2.8: Electric potential as a function of position in a circuit containing two capacitors, a battery, and a resistor. The values will change in the course of time, but the form of the diagram will basically remain the same.

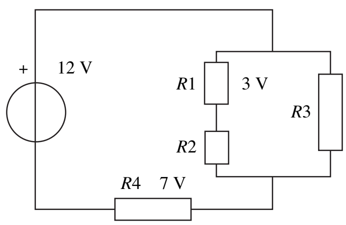

Figure 2.9: Sketch of an electric circuit having two branches, a battery (or power supply) and a number of resistors.

2.3 ELECTRIC POTENTIAL AND POTENTIAL DIFFERENCES (VOLTAGES)

26. Trace the electric potential in a circuit consisting of a real battery and a lamp. Sketch a level diagram such as that in Fig. 2.8. Identify voltages. What is the relation between all the voltages?

27. An electric motor contains coils of wire, and the wire has resistive properties. Is the voltage measured across the motor during operation equal to the voltage that is said to cause the production of heat?

28. A galvanic element shows a voltage of 1.5 V. How can a 12 V battery be made from several such elements?

29. A battery (12 V) is part of a closed electrical circuit. Attached to it is a resistor, a motor, another battery (5.0 V), and finally, a lamp. The voltages across the motor and the lamp are 6.0 V and 7.0 V.
    a. Sketch the potential-position diagram.
    b. What is the voltage across all the elements (with signs)?

30. Determine the missing voltage values in the electrical circuit of Fig. 2.9.

31. There is a voltage of 3.0 V along a uniform wire of 0.50 m length. What is the gradient of the electric potential along the wire in the direction of flow of charge? What is the sign of U?

2.4 ENERGY IN ELECTRIC PROCESSES

32. Consider an electric water pump. What is the meaning of "driving process?" What is the driven process? What does energy have to do with one process driving another? What is the meaning of electric power? Of hydraulic power?

33. Explain the role of energy in the operation of a battery. Explain terms such as driving and driven process, power, energy storage, etc.

34. Consider a battery driving an electric pump. What is the role of energy?

35. Describe releasing and binding (using) of energy in the electrical processes of (1) an immersion heater, (2) a battery and an electric motor. Draw the system diagrams. Which processes are coupled in these devices? Which process drives? Which one is driven?

36. A pump is driven by an electric engine. Together, the motor and the pump have an efficiency of 80%. One wishes to pump a water current of 0.020 m^3/s 10 m high. What must the electric current be for the drive to be at least 220 V? Does it need to be greater than 220 V?

37. A voltage of 210 V and a current of 4.0 A are measured in a running electric motor.
    a. What is the electric power of the motor?
    b. What is the released energy used for?

2.5 FLOW RELATIONS

38. Assume a type of conducting device for which the electric current doubles if the voltage across it is doubled. What is the characteristic diagram of such a device? How do you explain the meaning of conductance or resistance of the device using the characteristic I_Q–U_R diagram?

39. Metals are said to be ohmic conductors which are said to have linear characteristic curves: double the voltage across an ohmic conductor or resistor gives double the current through the device. The filament of a light bulb is a metal. Why is the characteristic of an incandescent lamp not linear?

40. Sketch the current-voltage characteristics for the following resistive elements:
    a. An element with constant resistance.
    b. An element similar to the one in a, but with a greater resistance value.
    c. The filament of a light bulb.

41. Derive the expressions for equivalent resistances of resistors in series or in parallel. Imagine two identical resistors. What is their equivalent resistance if they are connected in parallel?

42. A 100 W incandescent light bulb is connected to 220 V. What is the resistance of the filament? Is the resistance different if it is connected to 150 V?

43. What is the ratio of the resistance of copper wires having the same length where one has double the diameter of the other?

44. We have two equal amounts of a metal. Each is formed into a wire. The second wire is twice as long as the first. What is its resistance compared to the resistance of the first one?

45. At 20°C, platinum has a resistivity of $0.107 \cdot 10^{-6}$ Ω·m. What must the radius of a 0.50 m long platinum wire be so that it has a resistance of 20 Ω?

46. The temperature coefficient of the resistivity of platinum is $3.90 \cdot 10^{-3}$ K^{-1}. What is the resistance of the wire in Exercise 45 at 120°C?

47. The resistances of R1 and R2 in Fig. 2.9 are 20 Ω and 10 Ω.
    a. What is the total resistance in the branch with R1 and R2?
    b. What is R3 when the current through this branch is twice as great as in R1?
    c. What are the equivalent resistances for R1, R2, and R3?

48. What is the electric current through an immersion heater hooked up to 220 V having an electric power of 300 W? What is the thermal power of this device?

49. Two resistors with resistances of R_1 and R_2 are both connected to an ideal battery with a voltage of U_B.
    a. Find the electric power of all three elements.
    b. Why does energy conservation in this circuit lead to Kirchhoff's law?

50. Fig. 2.10 shows the measured I–U characteristic of a light bulb.
    a. Determine the resistance and the electric power in steps of 5 V.

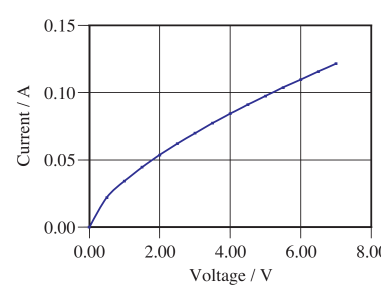

Figure 2.10: Characterisitc diagram of the filament of a light bulb.

b. Explain in words the meaning of the electric power of a filament.

c. The behavior of the temperature of the electrical resistance can be approximated using the quadratic formula

$$R(T) = R_{20°}\left(1 + \alpha\left(T - T_{20°}\right) + \beta\left(T - T_{20°}\right)^2\right)$$
$$\alpha = 4.11 \cdot 10^{-3}\,\mathrm{K}^{-1} \ , \quad \beta = 9.62 \cdot 10^{-7}\,\mathrm{K}^{-2}$$

Find the temperature of the filament.

d. How is the energy that is released from the filament transported away?

2.6 BATTERIES

51. You find 8.0 Ah written on a battery. What does this mean? A battery has 1.5 V written on it. What does this mean?

52. Why does the voltage measured across the terminals of a battery decrease if the electric current through the battery is increasing? How are voltage and current related?

53. Why is the current through a battery limited, even for load resistors with very small resistances (such as a simple short wire connecting the terminals)?

54. A lamp is connected to a 4.5 V battery. A voltage of 4.1 V is measured across the lamp. The voltages over the wires in the circuit are just about zero. What is going on?

55. The open-circuit voltage of a battery is 4.5 V. When it is connected to a small motor, it shows 4.0 V. The current measured in the circuit is 0.60 A.
    a. At what rate is energy released in the battery?
    b. What happens with the energy released by the battery? What is the energy current emitted by the battery along with the electric charge?
    c. What is the electric power of the motor?

56. Why is there a maximum of the power of a battery in operation? What is its power for either maximum voltage or maximum current?

57. Estimate the maximum power of the photovoltaic array in Fig. 2.11 for an insolation of 400 W.

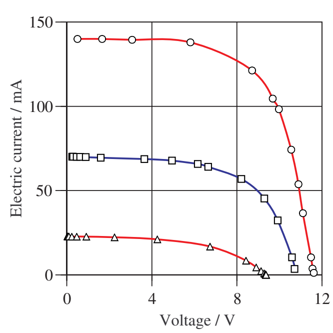

Figure 2.11: Characteristic curves of a small array of 21 solar cells in series exposed to the sun and the sky on three different winter days. The insolation was 400, 200, and 60 W/m^2 for the three curves (from the top).

2.7 STORING CHARGE

58. Consider a capacitor having constant capacitance. What is the form of the capacitive characteristic?

59. Explain the meaning of electric capacitance.

60. Imagine a capacitor being discharged in a simple circuit. What kind of data should be taken to derive the capacitive relation of the capacitor? How do you determine it?

61. What is the charge of a 10 μF capacitor which has been charged to 100 V?

62. A capacitor having a capacitance of 100 μF is charged to a voltage of 10 V. How much charge and energy is stored?

63. One wishes to store at least 1.0 J of energy in a capacitor that can be charged to 200 V. How great must its capacitance be made for this?

64. Produce a word model that explains the charging or discharging of a capacitor in a simple circuit having a resistor (Fig. 2.12). Do the same for discharging of a tank containing oil through a pipe at its bottom. Compare the explanations you use in the electric and in the hydraulic cases.

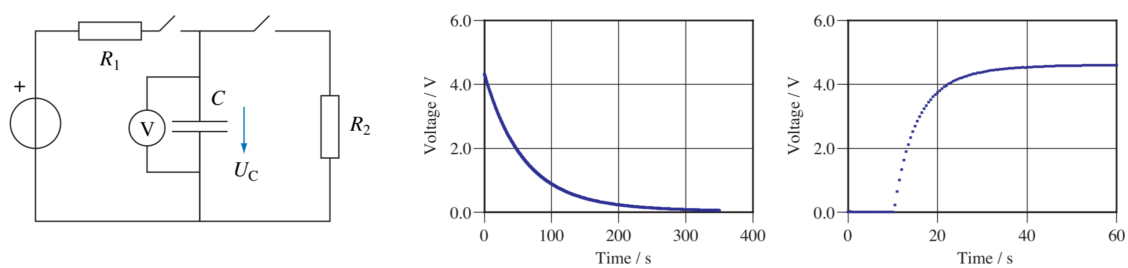

Figure 2.12: Discharging and charging of a simple capacitor. Left: Diagram of a circuit that allows for charging and subsequent discharging (R_1 = 9.8 kOhm, R_2 = 108 kOhm). Center: Voltage across the capacitor as a function of time, as it discharges. Right: Voltage across capacitor during charging.

65. Does it make more sense to compare a parallel plate capacitor with a water tank or a U-pipe? Which quantities do you compare? What is the capacitance of a U-pipe compared to normal pipe of equal thickness? What does this mean for an electric parallel plate capacitor?

66. A capacitor with a capacitance C is charged by an ideal battery with a voltage of U_B over a resistor having resistance R.
    a. What is the voltage across the capacitor at the moment the current has half of its initial value?
    b. What is the energy current emitted by the battery at the beginning? Where does the energy go?
    c. What is the energy current emitted by the battery when the current is only half what it was at the beginning? Where does the energy go?

2.8 DYNAMICAL MODELS

67. In what sense can we say that a model such as the one in Fig. 2.13 explains a system?

68. Explain the meaning of the structure of stocks (rectangles) and flows (thick arrows) in the system dynamics diagram of Fig. 2.13. Why is there only a single stock? Why are there two flows connected to the stock for Q?

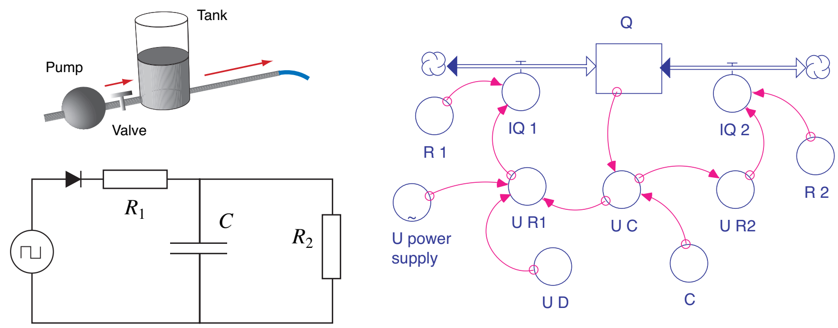

Figure 2.13: A system dynamics model diagram for a system equivalent to a hydraulic windkessel.

69. In Fig. 2.13, UD symbolizes the voltage across the diode which we set to a constant value independent of the current. How is UR1 calculated?

70. To what percentage of the final level does the level on the right in the diagram of Fig. 2.14 rise in one time constant?

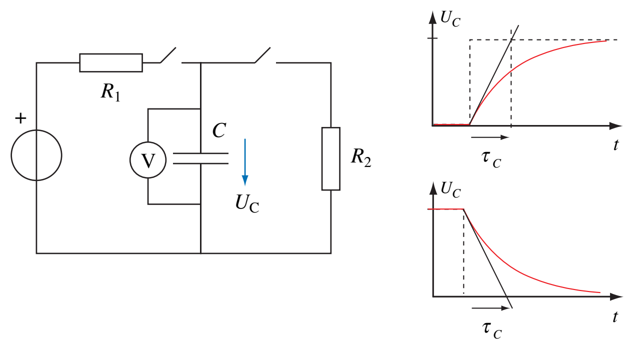

Figure 2.14: Discharging or charging of capacitors having constant capacitance through resistors having constant leads to exponentially changing functions. The initial rate of change is used to define the time constant of the system.

71. Estimate the time constant of the circuit in Fig. 2.2.

72. Measure the time constants in Fig. 2.12. Calculate the capacitance of the capacitor for both determinations of time constants.

73. A 1.0 mF capacitor discharges through a 1000 Ω resistor.
    a. How long does it take for the charge to reduce to 1/e of the initial value?
    b. How long does this take for the voltage of the capacitor and the current in the circuit to reduce to 1/ of their initial values?
    c. How long does it take until only half of the charge remains in the capacitor?