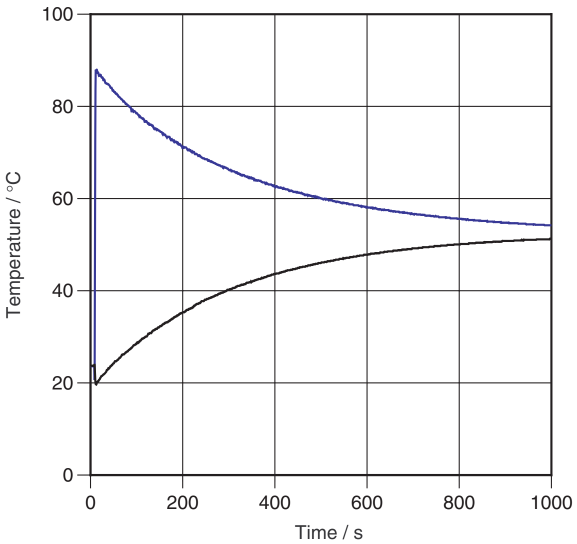
PHYSICS EXAM

1. Water is vaporized in a container which is open at the top, but otherwise insulated. We observe that the mass of the water decreases linearly with time when it is heated at constant electric power. Does this mean that the change of entropy in the water/steam system is proportional to the evaporated mass? Explain.

2. Glycol has an energy capacity which, between 0 °C and 100°C, is approximately proportional to the absolute temperature. Sketch the pertaining *s-T* diagram (specific entropy as a function of the temperature). Give the necessary explanations.

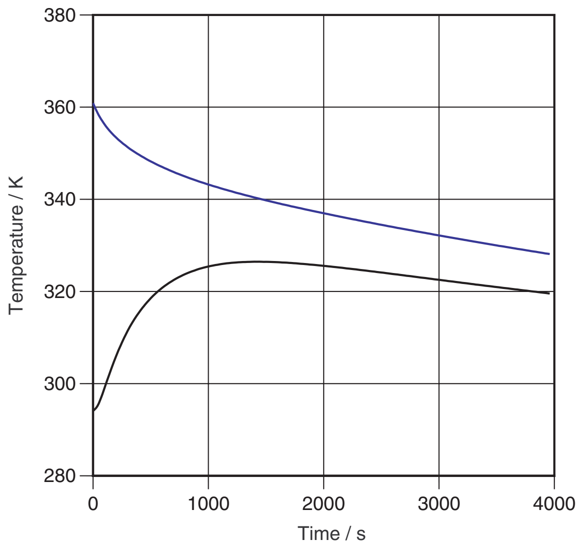
3. We have two containers in thermal contact and containing the same amount of water. In one of them is hot water, in the other cold water. Both of these containers are perfectly insulated to the surroundings. Describe what happens with the entropy and energy of the two bodies of water (individually and together), and what happens with the temperature of the two bodies.

4. Two amounts of water of 951g each are in thermal contact through a thin metal wall (but otherwise perfectly insulated). Determine for $t = 400$ s:
   a. the rate of change of temperature of the warm body;
   b. the energy flow out of the warm water;
   c. The entropy flow out of the warm water;
   d. The rate of entropy production;
   e. Should the rate of change of temperature for the cold water equal that of the warm water (apart frm the sign)? Explanation.



5. Water having a mass of 300 g is cooled in a PVC container (mass 510 g). Take the specific energy capacity of PVC to be 1000 J/(K·kg). The air temperature is 21°C. The temperature curves in the diagram are for the water and the middle point of the container wall. The wall is 1.50 cm thick.

   The lid and the bottom of the container are perfectly insulated. The outer surface of the container is 0.031 m^2. The convective heat transfer from the water to the container is assumed to be perfect. Between the container and the air, the heat transfer coefficient is 10 W/(K·m^2). Determine for $t = 2000$ s
   a. the rate of change of temperature of the water;
   b. the rate of change of temperature of the container wall;
   c. the energy current out of the water;
   d. the rate of change of energy of the container;
   e. the energy flow from the container to the air;
   f. the temperature at the outer surface of the container;
   g. the thermal conductivity of PVC.



SOLUTIONS

1. Water is vaporized in a container which is open at the top, but otherwise insulated. We observe that the mass of the water decreases linearly with time when it is heated at constant electric power. Does this mean that the change of entropy in the water/steam system is proportional to the evaporated mass? Explain.

SOLUTION

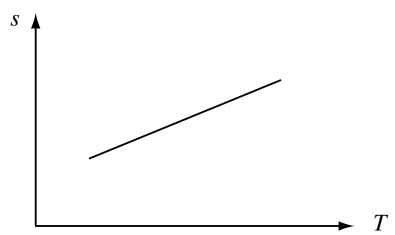
Yes. The mass of steam generated is proportional to time. The amount of entropy added to the system also increases linearly with time. This is because of

$$I_S = \frac{1}{T_{evap}} I_{W,th} = const.$$

Remember that the temperature of evaporation stays constant.

2. Glycol has an energy capacity which, between 0 °C and 100°C, is approximately proportional to the absolute temperature. Sketch the pertaining *s-T* diagram (specific entropy as a function of the temperature). Give the necessary explanations.

SOLUTION



$$c = aT \quad , \quad a = const.$$
$$\Rightarrow \quad k = c/T = a$$
$$\Rightarrow \quad s = s_o + a(T - T_o)$$

3. We have two containers in thermal contact and containing the same amount of water. In one of them is hot water, in the other cold water. Both of these containers are perfectly insulated to the surroundings. Describe what happens with the entropy and energy of the two bodies of water (individually and together), and what happens with the temperature of the two bodies.

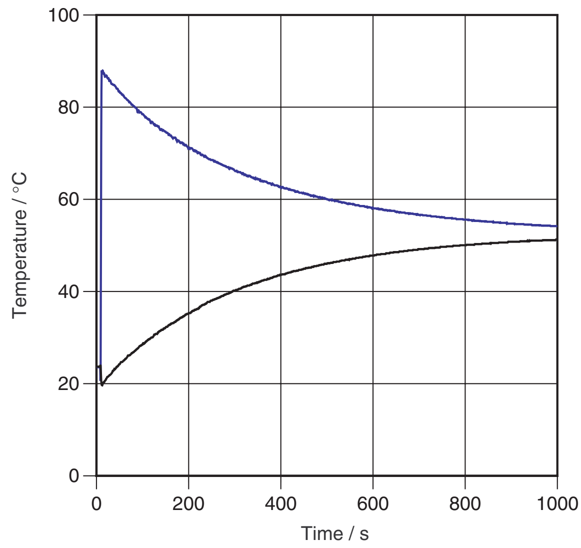
SOLUTION

Table 1: Entropy and Energy

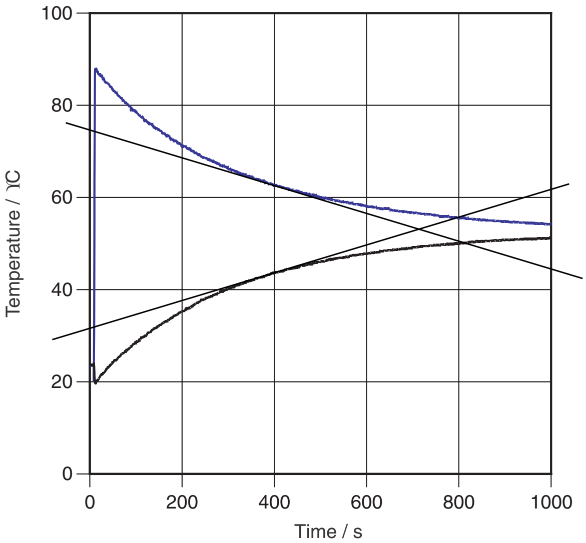
	Hot body	Cold body	Both
Entropy	decreases	increases	increases because of production
Energy	decreases	increases	is constant

Temperature: Temperature of the hot body decreases, temperature of cold body increases; temperature reaches the same final value in both bodies.

4. Two amounts of water of 951g each are in thermal contact through a thin metal wall (but otherwise perfectly insulated). Determine for $t = 400$ s:
   a. the rate of change of temperature of the warm body;
   b. the energy flow out of the warm water;
   c. The entropy flow out of the warm water;
   d. The rate of entropy production;
   e. Should the rate of change of temperature for the cold water equal that of the warm water (apart frm the sign)? Explanation.



SOLUTION



a. Slope of tangent to first curve at t = 400 s: dT/dt = – 30 K / 1000 s = – 0.030 K/s.

b.

$$\frac{dW}{dt} = I_W \quad , \quad \frac{dW}{dt} = C\dot{T}$$

Therefore

$$I_W = C\dot{T} = 0.951\text{kg} \cdot 4200\text{J}/(\text{K} \cdot \text{kg}) \cdot (-0.030\text{K/s}) = -120\text{W}$$

c.

$$I_S = \frac{I_W}{T_1} = \frac{-120\text{W}}{(62.7 + 273)\text{K}} = -0.357\text{W/K}$$

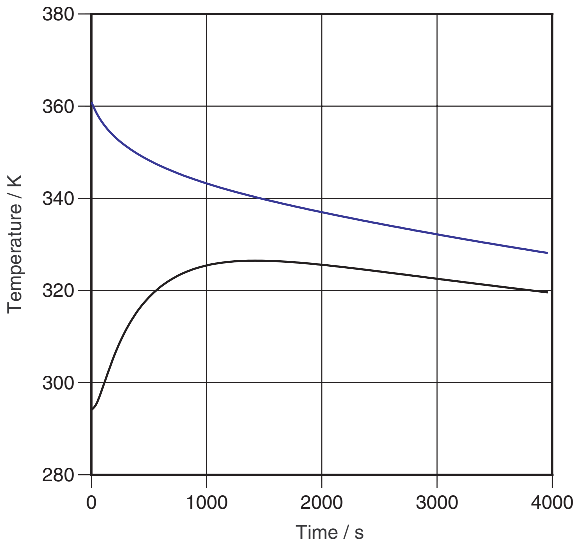
d.

$$\Pi_S = \frac{1}{T_2}|I_S|(T_1 - T_2) = \frac{1}{(43.5 + 273)\text{K}}(0.357\text{W/K})(62.7 - 43.5)\text{K} = 0.022\text{W/K}$$

e. Yes. The rate of change of energy of the second body is the negative of the value for the first. Since the energy capacitance of water is constant, the rates of change of temperature of the two bodies must be equal (but of opposite sign):

$$\dot{W}_1 = -\dot{W}_2 \quad \Rightarrow \quad C_1\dot{T}_1 = -C_2\dot{T}_2$$
$$C_1 = C_2 \quad \Rightarrow \quad \dot{T}_1 = -\dot{T}_2$$

5. Water having a mass of 300 g is cooled in a PVC container (mass 510 g). Take the specific energy capacity of PVC to be 1000 J/(K·kg). The air temperature is 21°C. The temperature curves in the diagram are for the water and the middle point of the container wall. The wall is 1.50 cm thick. The lid and the bottom of the container are perfectly insulated. The outer surface of the container is 0.031 m^2. The convective heat transfer from the water to the container is assumed to be perfect. Between the container and the air, the heat transfer coefficient is 10 $W/(K \cdot m^2)$. Determine for t = 2000 s
   a. the rate of change of temperature of the water;
   b. the rate of change of temperature of the container wall;
   c. the energy current out of the water;
   d. the rate of change of energy of the container;
   e. the energy flow from the container to the air;
   f. the temperature at the outer surface of the container;
   g. the thermal conductivity of PVC.

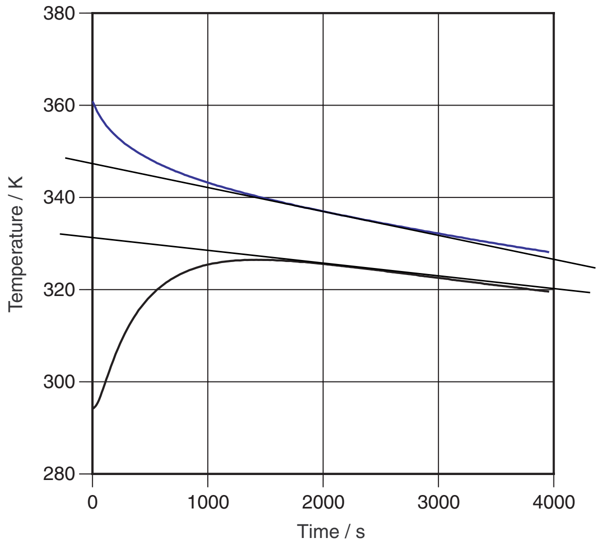


SOLUTION

a. and b. (see fugure below) The rates of change of temperature are obtained from the slopes of the tangents to the curves:

dT_1/dt = – 20.7 K / 4000 s = – $5.18 \cdot 10^{-3}$ K/s

dT_2/dt = – 11.4 K / 4000 s = – $2.85 \cdot 10^{-3}$ K/s



c. The rate of change of temperature determines the rate of change of energy of a body. Since there is only one energy current out of the water (that is our assumption), we have:

$$I_W = C\dot{T}_1 = 0.300\text{kg} \cdot 4200\text{J}/(\text{K} \cdot \text{kg}) \cdot (-0.0052\text{K/s}) = -6.53\text{W}$$

d. The rate of change of temperature determines the rate of change of energy of the body:

$$\dot{W}_2 = C_2\dot{T}_2 = 0.510\text{kg} \cdot 1000\text{J}/(\text{K} \cdot \text{kg}) \cdot (-0.00285\text{K/s}) = -1.45\text{W}$$

e. Knowing the rate of change of energy of the container wall, and the energy current into the wall (which is equal to the current out of the water), we calculate the current out of the wall from the law of balance of energy:

$$\dot{W}_2 = I_{W,in} + I_{W,out} \quad \Rightarrow \quad I_{W,out} = \dot{W}_2 - I_{W,in} = -1.45\text{W} - 6.53\text{W} = -7.98\text{W}$$

f. Knowing the energy current into the air we can calculate the temperature difference across the convective layer at the surface of the container. This yields the temperature at the surface of the container:

$$I_{W,th} = G_W\left(T_{c,surface} - T_a\right) = Ah\left(T_{c,surface} - T_a\right) = 0.031\text{m}^2 \cdot 10\text{W}/(\text{K} \cdot \text{m}^2) \cdot \left(T_{c,surface} - 294\text{K}\right)$$

$$\Rightarrow \quad T_{c,surface} = 294\text{K} + \frac{7.98\text{W}}{0.31\text{W/K}} = 320\text{K}$$

g. Again using the energy current from the center of the container wall to the air we calculate the temperature difference from the center of the wall to its surface (with the help of the conductive resistance):

$$I_{W,th} = G_{W,cond}\left(T_2 - T_{c,surface}\right) = Ah_{cond}\left(T_2 - T_{c,surface}\right) = A\frac{\lambda}{\Delta x}\left(T_2 - T_{c,surface}\right)$$

$$\Rightarrow \quad \lambda = \frac{\Delta x I_{W,th}}{A\left(T_2 - T_{c,surface}\right)} = \frac{0.0075\text{m} \cdot 7.98\text{W}}{0.031\text{m}^2 \cdot (325.7\text{K} - 320\text{K})} = 0.34\text{W}/(\text{K} \cdot \text{m})$$

[Remark: This value is more than twice as large as it should be. The error is with an assumption in our model. There is heat loss through the lid (and some through the bottom) of the container, making the energy current through the mantle to the air 6 W (not 8 W as in our calculation). This in turn makes the surface temperature 314 K (not 320 K). With these corrections we get 0.12 W/(K·m) for the conductivity of the material (compared to a value of 0.15 W/(K·m) found in tables).]