5/21/2002: Radioactive Decay and Neutron Activation Lab
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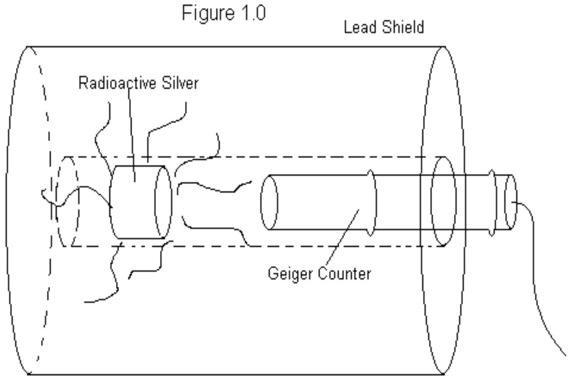
Description:

In 1979 the Nobel Prize in physics was awarded to three men. Glashow, Salam and Weinberg for their work in proving how the weak force interaction causes such phenomena as beta decay. Their work helped pave the way for the nuclear physics that has occurred in the past two decades. This lab deals with not only beta decay processes governed by the weak force, but also alpha decay, alpha capture and neutron activation.

Silver metal is made up of two stable Isotopes. These isotopes are ^{109}Ag and ^{107}Ag, which have a relative abundance of 48.6% and 51.4% in a sample of silver. By bombarding silver nuclei with neutrons it is possible to activate them into a state of radioactivity in which they will beta decay to two isotopes of Cd.

Figure 1.0



In order to bombard our silver sample we placed it in range of a source consisting of radium and beryllium. The radium decays to give up an alpha particle which causes the beryllium to eject a neutron from its nucleus. The following reactions took place to give up neutrons:

Eq. 1.0 $\quad ^{226}\mathbf{Ra} \Rightarrow \mathbf{Rn} + \alpha$

Eq. 1.1 $\quad ^{6}\mathbf{Be} + \alpha \Rightarrow {}^{12}\mathbf{C} + \mathbf{n}$

The neutrons freed in the previous reactions were absorbed by the stable isotopes within our silver sample to activate them by the following reactions:

Eq. 1.2 $\quad ^{109}\mathbf{Ag} + \mathbf{n} \Rightarrow {}^{110}\mathbf{Ag} + \gamma$

Eq. 1.3 $\quad ^{107}\mathbf{Ag} + \mathbf{n} \Rightarrow {}^{108}\mathbf{Ag} + \gamma$

Once our sample had been completely activated it was removed from the source and allowed to beta decay by the processes given in Equations 1.4 and 1.5. Using a Gieger counter encased in a lead shield (figure 1.0) we were able to take a reading of the emitted beta particles. We were given that the half-lives for these processes were 24.4 seconds and 145 seconds respectively.

Eq. 1.4 $\quad ^{110}\mathbf{Ag} \Rightarrow {}^{110}\mathbf{Cd} + \mathbf{e}^{-} + \nu^{-}$

Eq. 1.5 $\quad ^{108}\mathbf{Ag} \Rightarrow {}^{108}\mathbf{Cd} + \mathbf{e}^{-} + \nu^{-}$

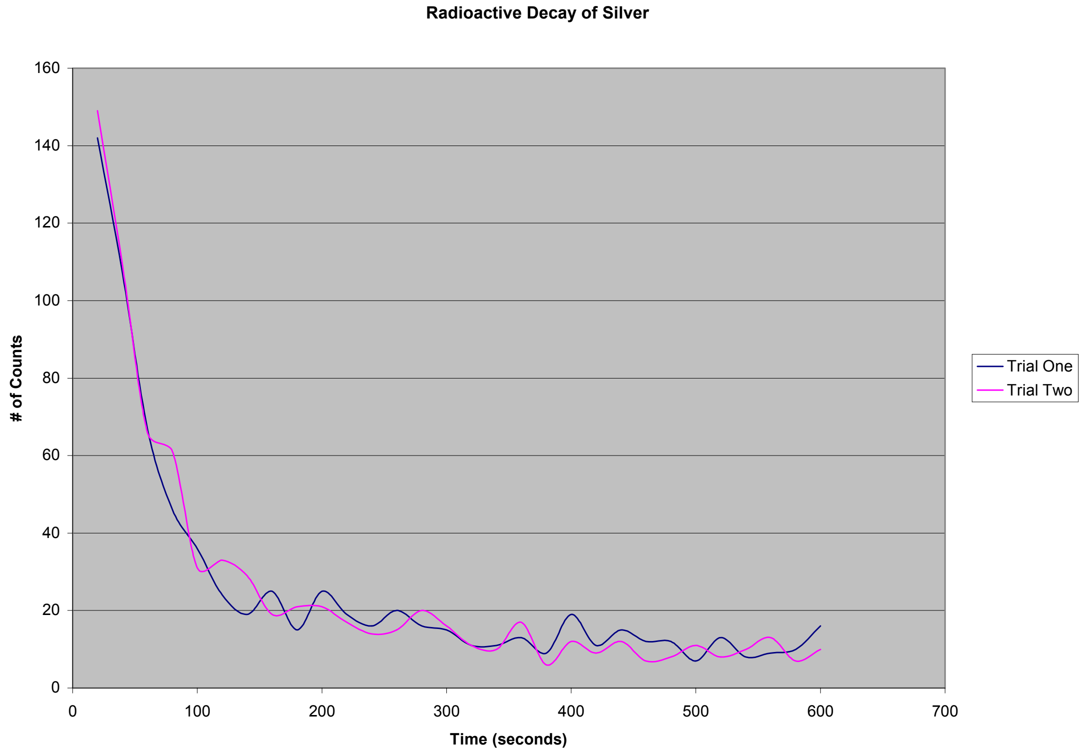
We also measured a background count rate when there was no sample within range of our Geiger counter. We took two sets of background counts and two trials with our radioactive sample of Silver.

All relevant data is recorded in the data section. Exercises 1 through 3 are found in the data section. Analysis questions sections I and II are recorded in the analysis section. Conclusions are drawn in the discussion section.

Data:

time (sec)	Bkgrd 1	Bkgrd 2	Trial 1	Trial 2	Bkgrd Avg.
20	4	8	142	149	6
40	8	7	107	109	7.5
60	8	7	67	66	7.5

80	**8**	**6**	**46**	**61**	**7**
100	**9**	**13**	**36**	**31**	**11**
120	**4**	**9**	**24**	**33**	**6.5**
140	**12**	**9**	**19**	**29**	**10.5**
160	**9**	**4**	**25**	**19**	**6.5**
180	**6**	**7**	**15**	**21**	**6.5**
200	**9**	**5**	**25**	**21**	**7**
220	**4**	**5**	**19**	**17**	**4.5**
240	**7**	**5**	**16**	**14**	**6**
260	**10**	**8**	**20**	**15**	**9**
280	**9**	**11**	**16**	**20**	**10**
300	**8**	**10**	**15**	**16**	**9**
320	**6**	**5**	**11**	**11**	**5.5**
340	**5**	**5**	**11**	**10**	**5**
360	**8**	**9**	**13**	**17**	**8.5**
380	**7**	**6**	**9**	**6**	**6.5**
400	**5**	**2**	**19**	**12**	**3.5**
420	**10**	**6**	**11**	**9**	**8**
440	**7**	**6**	**15**	**12**	**6.5**
460	**7**	**7**	**12**	**7**	**7**
480	**10**	**9**	**12**	**8**	**9.5**
500	**7**	**5**	**7**	**11**	**6**
520	**11**	**7**	**13**	**8**	**9**
540	**12**	**3**	**8**	**10**	**7.5**
560	**11**	**5**	**9**	**13**	**8**
580	**6**	**4**	**10**	**7**	**5**
600	**10**	**8**	**16**	**10**	**9**
		Average:	**25.6**	**25.73333**	**7.3**
	Standard Deviation:		**29.42176**	**31.85304**	**1.808028**



Exercise 1:

We can assume from our data that the amount of radioactive particles in the sample decreases at a rate proportional to number present in the sample at a given time. That is to say:

$dx/dt = -kx$

We can solve this as a separable first order differential equation where x is the concentration ranging from A_0 to $A(t)$ and t is the time ranging from zero to t.

$$\int dx/x = \int -k\,dt \quad \Leftrightarrow \quad (\ln x)[A_0 < x < A] = (-kt)[0 < t < t]$$

$$\ln (A(t)/A_0) = -kt \quad \Leftrightarrow \quad A(t) = A_0 e^{-kt}$$

This last equation gives us a standard form for any decreasing exponential system. By inputting some of the values we know we can solve for the half-life as a function of the decay constant k. If we are at t = one half-life we know that the ratio of A to A_0 has to be equal to ½. Thus we can state the following:

$$½ = e^{-kt} \quad \Leftrightarrow \quad -kt_{1/2} = \ln (½) \quad \Leftrightarrow \quad t_{1/2} = .693/k$$

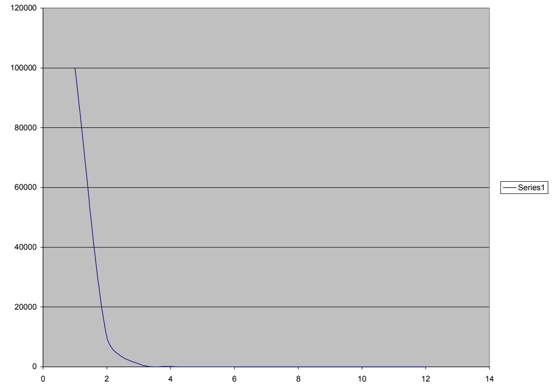
Exercise 2:

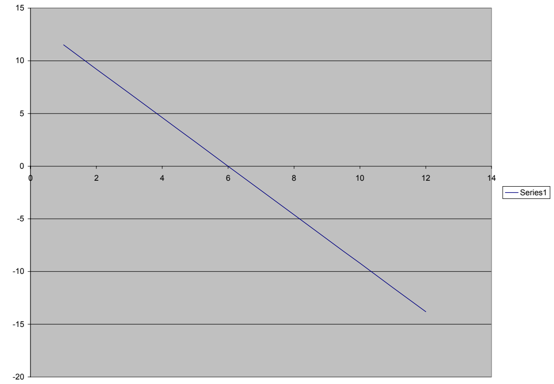
Both the concentrations of decaying particles and the activity of the sample follow the exponential that we have discovered above. So if we use the same logic that we used in the previous exercise we can determine the time that it would take for the activity to decrease to 1/8 the original activity.

After one half-life there will be ½ of the original activity left. After another half-life there will be half of what is left, or ¼ the original activity. And after a third half-life there will be 1/8 of the activity sample left.

Exercise 3:

If we plot the concentration of radioactive particles, or the activity versus time we should get decreasing exponential functions. If we plot the natural log of the functions we should get decreasing linear functions. Here are both functions for a sample set of data:





time	N(t)	ln (N(t))
1	100000	11.51293
2	10000	9.21034
3	1000	6.907755
4	100	4.60517

time	N(t)	ln (N(t))
5	10	2.302585
6	1	0
7	0.1	-2.30259
8	0.01	-4.60517

time	N(t)	ln (N(t))
9	0.001	-6.90776
10	0.0001	-9.21034
11	0.00001	-11.5129
12	0.000001	-13.8155

Using the slope formula for the linear graph we can see that the slope of that line is equal to the decay constant in the aforementioned formula.

$m = [\ln(A) - \ln(A_0)] / (t - 0) \quad \Leftrightarrow \quad m = \ln(A/A_0) / t$

$m = -kt/t \quad \Leftrightarrow \quad m = -k$

So we can easily find the half-life of the sample by using the equation above:

$t_{1/2} = 0.693/ k$

Analysis:

Section I

1. $n_{ave} = 1/N(\Sigma\, n_i)$ = 7.9 for trial one and 6.7for trial two.
   $s = \sqrt{[1/N\text{-}1(\Sigma\,(n_i - n_{ave})^2]}$ = 2.294671 for trial one and 2.409035 for trial two.
2. If $N \rightarrow \infty$, $s \rightarrow \sigma$, and $n_{ave} \rightarrow \mu$. If $\sigma^2 = \mu$, then we should have a reasonably good approximation of this in our values of s and n_{ave}.

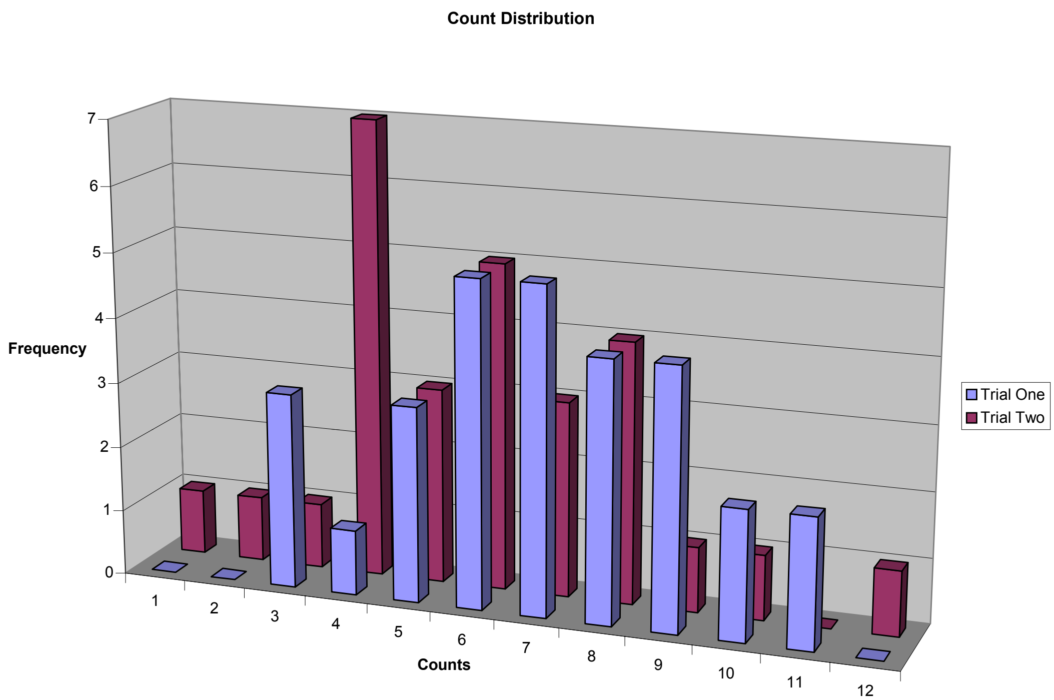
time	trial one	trial two
100	37	41
200	40	34
300	38	39
400	31	27
500	41	33
600	50	27
Average	39.5	33.5
Std Dev	6.220932	5.85662

$s_1^2 = 5.26$ and $s_2^2 = 5.80$ These values fall somewhat short of the expected values due to the small number of data points taken in the sample.

3. If the relationship given in part three of section one in the lab manual is correct then $(2.294671*\sqrt{(5)}) \approx 6.220932$. In fact it is closer to 5.024. Once again this is in large part due to the fact that the sample space is so small

4. I have chosen to change this question a little to better suit, in my opinion, the data and show the normal distribution better. I have taken both background readings and counted the frequencies of each number of counts occurring and plotted this set of data as two histograms. As can be seen in the below figure

there are some large abnormalities in the data. Once again these “bumps” would smooth out if we had a larger number of counts to choose from.



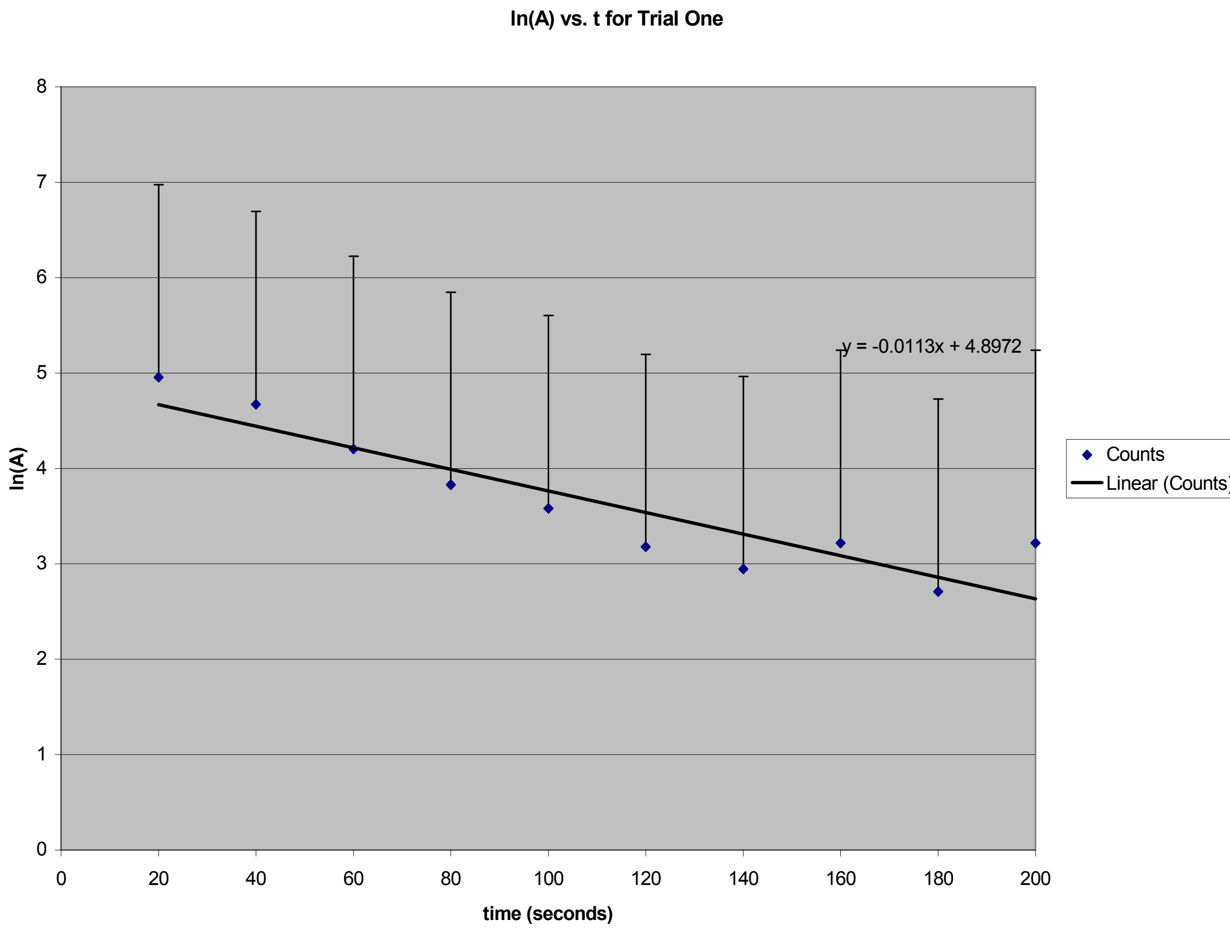
5. If we take, for trial one, the standard deviation to be roughly 2.3 and the mean value to be roughly 8 we should see 67% of the counts fall between 5.7 and 10.3 counts. For trial one we see that there are 169/258 counts in this region and thus 66% of the counts are within one standard deviation. We should also see roughly 94% of the counts within 2 standard deviations (between 3.4 and 12.6). This works out to be 100% of the counts, this might work out better if there were a greater number of data points in the sample.
6. The best value for an average would be the average of all 60 of the background counts. This turns out to be 7.543.

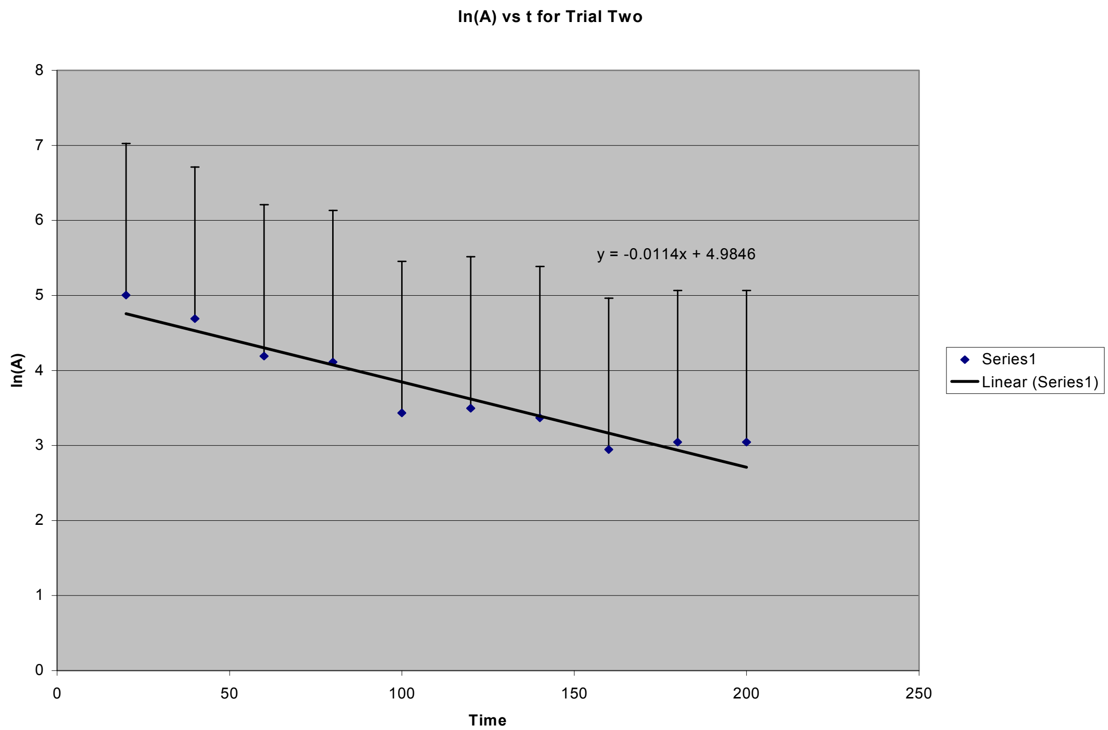
Section II

1. The half-lives of the two isotopes of silver are 24.4 seconds for ^{110}Ag and 145 seconds for ^{108}Ag. By the time ^{108}Ag has undergone one half-life, ^{110}Ag has undergone almost six (there is roughly 2% of the original sample left). If we were to look at the rates of decay for these two isotopes as a superposition of two straight lines the relative effect of the ^{110}Ag would drop off very quickly in comparison to the ^{108}Ag. This phenomenon can be seen especially well when we look at the decay constants for the two isotopes.
   $k = .693/t_{1/2}$ $k_{108} = .693/145\ s = 4.78E\text{-}3\ s^{-1}$ $k_{110} = 2.84E\text{-}2\ s^{-1}$
   As is clearly seen k_{110} is a whole order of magnitude larger than k_{108}. This is why as t gets large the ^{110}Ag sample does not effect the superposition curve. It might be going too far to say that as t goes to infinity the slope of the superposition curve goes to the slope of just the decay of ^{108}Ag. Clearly the

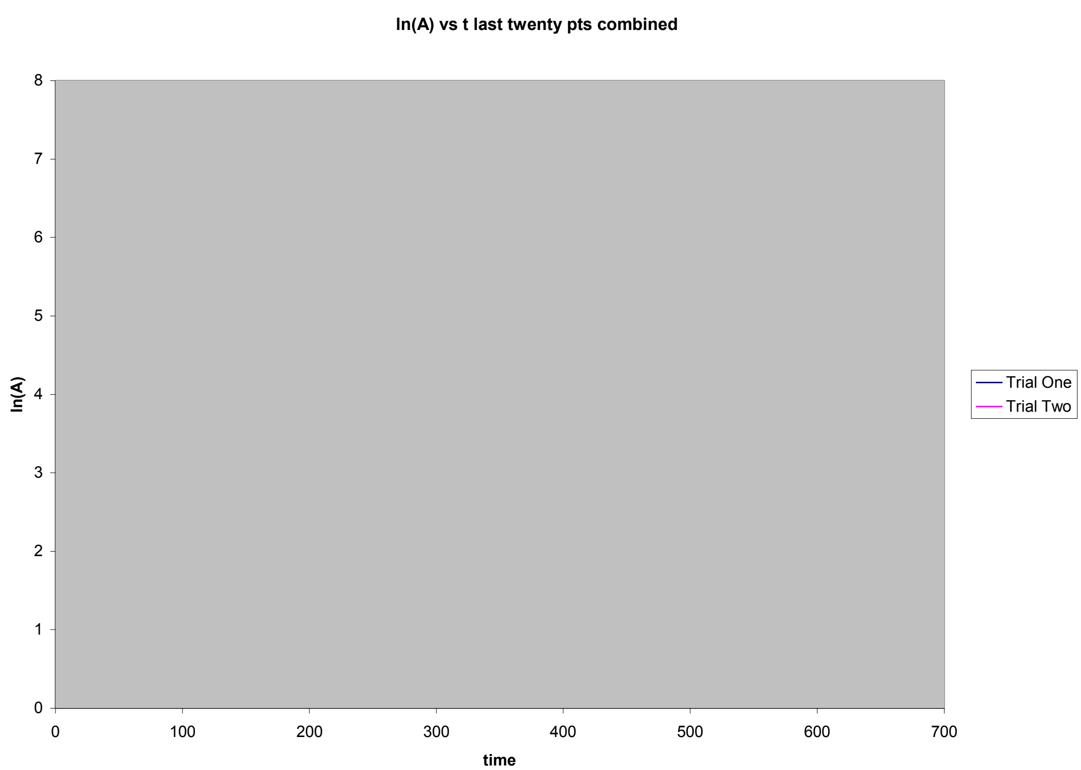
equation for the activity goes to zero as t goes to infinity. But if we restrict t to be large, but not infinity we see that the presumption is correct.

2. This will be omitted to save time
3. As was stated earlier the plot of the ln(A) versus t should be a superposition of two straight lines. The first of these lines has a slope roughly equal to the decay constant of ^{110}Ag, and the second has a slope roughly equal to the decay constant of ^{108}Ag. For very small values the superposition of these lines is very close to the first line, and for large values it is very close to the second line.
4. For the error we used the natural log of 7.543 (the average obtained in part I). It occurred to me that we would never observe error on the negative side, that is to say background radiation would never allow us to see fewer counts than were actually there so we only put error bars on the positive side.




ln(A) vs t for Trial Two
y = -0.0114x + 4.9846
ln(A)
Time
Series1
Linear (Series1)
0
1
2
3
4
5
6
7
8
0
50
100
150
200
250

5.


ln(A) vs t last twenty pts combined
ln(A)
time
Trial One
Trial Two
0
1
2
3
4
5
6
7
8
0
100
200
300
400
500
600
700

6. $m = (y_2-y_1)/(x_2-x_1)$ $m_\infty = - k_{108} = .00205$
   $m_0 = -0.01684$ $m_\infty = -.00205$ $k_{108}N_0^{108} = (.00205*146) = .2993$
   $m_0 = -0.01684 = -k_{110}[1-(k_{108}N_0^{108}/k_{110}N_0^{110})]$

Discussion:

This lab was a good demonstration of how statistics is a huge part of nuclear physics. It was interesting to see how the half-lives of the isotopes affected each other in the decay. A simple yet intricate chain of events takes place to make this happen. I think this was one of my favorite labs of the term.