INVESTIGATION 7

# MOMENTUM AND MOMENTUM TRANSPORT IN COLLISIONS

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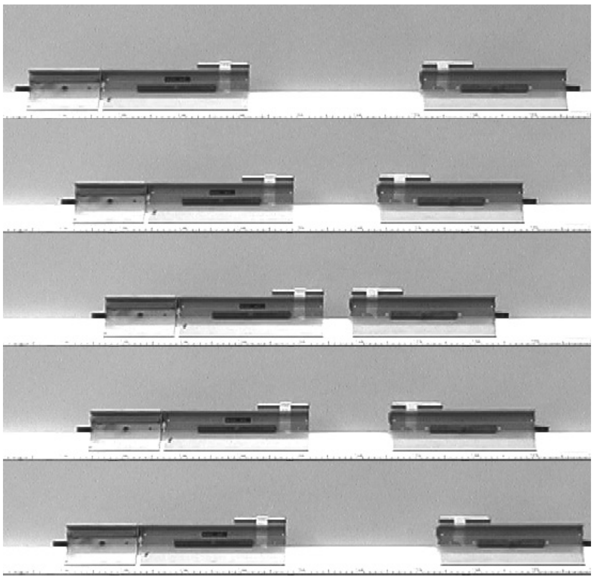


In this investigation, we will study the collision of two gliders using repelling magnets. The gliders move on an air track in a single spatial dimension. Position data can be extracted from a movie. A dynamical model will be constructed with several goals in mind. Above all, we want to learn that simple imaginative concepts suffice to construct a basic understanding of motion phenomena. Second, we will be able to produce a law of mechanical interaction which we do not take from books. Third, we will compare model simulations to the data taken from the experiment to determine parameters of the model. Finally, energy quantities will be added to our model to demonstrate how we can understand energy in mechanical systems.

## 7.1 EXPERIMENT AND DATA

Two gliders move on a horizontal air track. The air cushion ensures that the gliders move almost without friction. One glider is standard size, the second is composed of a standard one plus a smaller one (making for a heavier glider). Two magnets are mounted on the gliders (Fig. 7.1) in such a way that they repel each other when they come close. The weights (masses) of the gliders and magnets are determined with the help of a scale. A standard glider has a mass of 503 g, that of the smaller glider is equal to 250 g, and the mass of a magnet equals 115 g. The length of a magnet is 9.1 cm. The gliders are made to move toward each other which leads to a collision. A movie of the resulting motion is taken for analysis.

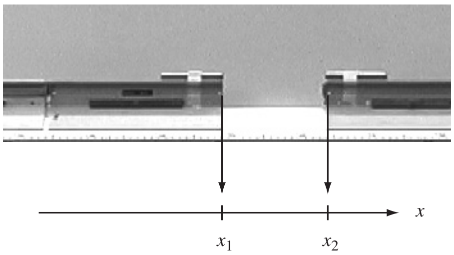



**Figure 7.1:** Two different gliders with magnets mounted on them on a horizontal air track. Positions of the fronts of the gliders are measured as functions of time from the frames of a movie.

### 7.1.1 Taking position data

The positions of the gliders are determined from the movie. (Fig. 7.1). Frame by frame, the positions of the facing fronts of the gliders are measured relative to the measuring tape attached to the air track (alternatively, pixel values can be obtained with the help of video analysis tools; the pixel values are then converted to actual distances). In Fig. 7.2, the positions of the faces of the two gliders and their distance are shown as functions of time.

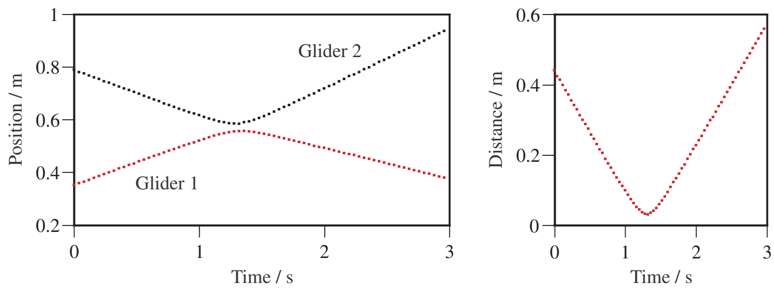



**Figure 7.2:** Positions of the fronts of the two gliders (left) and distance between the gliders (right) as functions of time.

### 7.1.2 Determining speed from position data

If we know the position of an object as a function of time, its speed can be determined easily as the rate of change of the position. If we do this for the data shown in Fig. 7.2, we get the speeds of the two gliders as functions of time. The result can be seen in Fig. 7.3.

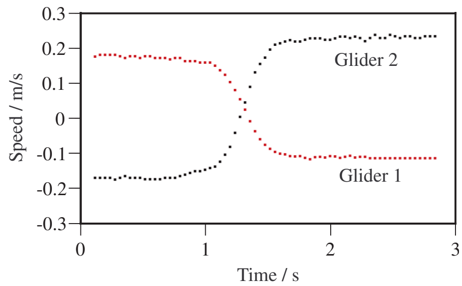



**Figure 7.3:** Speeds of the two gliders derived from the position data in Fig. 7.2. The average rate of change of position over two time steps ($\Delta t = 0.04$ s) is taken and then the results is smoothed by using centered moving averages over 5 points.

Notice that both gliders reverse their directions of motion during the collision so the signs of the speeds are reversed. As long as the gliders are far enough apart, their speeds are nearly constant. They change during the collision. The change of speed for Glider 1 is larger than the corresponding change for Glider 2.

## 7.2 MOMENTUM AND SPEED

How can motion phenomena be understood? It will be shown that we need mostly two concepts: *quantity of motion* and *speed*, to describe and formally model phenomena involving translational motion. [Translation has to be distinguished from rotation. In rotation, a body stays in its place while its parts move about an axis through its center.]

### 7.2.1 Speed as the level of motion

Visually speaking, motion consists of the change of position which happens at various speeds. So speed appears to be the quantity we can use to describe and possibly understand motion. However, at closer look, it becomes apparent that another quantity is needed to understand what is happening in a collision.

It is common to speak of collisions as a process during which the participating bodies exchange something. Speed does not serve this idea well: note that Glider 2 appears to "receive" more speed than Glider 1 lost.

Other phenomena tell us that speed has a different conceptual character than a quantity that is exchanged in collisions. Consider the following process. A block is pushed strongly onto a waiting cart (Fig. 7.4). As the block slides onto the cart, it sets the latter in motion. This is so because of friction. As the process continues, the block is slowing down relative to the cart which is speeding up relative to the ground. Finally, the block will not move any longer relative to the cart and it moves along with the cart: *both bodies have attained the same spee*d.

We have seen this behavior before in several completely different types of phenomena: interacting elements attain the same level or potential if we wait long enough for equilibrium to set in. This indicates that we should think of *speed as the level or potential of motion*.

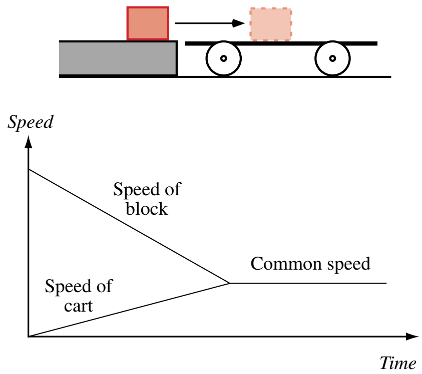



**Figure 7.4:** A block slides onto a cart setting the cart in motion. This process goes on until block and cart have the same speeds.

### 7.2.2 Momentum as the quantity of motion

We will have to look elsewhere for the quantity of motion. The process discussed above (Fig. 7.4) tells us that we do not have to go far: the moving block has a quantity of motion. Because of the interaction of the block and the cart, it communicates part of its quantity of motion to the cart which will then move as well. The quantity of motion lost by the block is communicated to the cart which means that we may assume the total quantity of motion of block and cart to remain fixed during the interaction.

The quantity of motion (introduced by Newton) is what we now call *momentum*. Momentum is conceived of as the quantity of motion having properties similar to those quantities communicated (transferred) in other phenomena: volume in fluid processes, charge in electricity, entropy in thermal phenomena, etc. Momentum is contained in moving bodies, and it can be transferred from body to body if they interact mechanically. Barring more detailed knowledge, we will start our investigation by assuming that momentum can neither be created nor destroyed. [We know that bodies can easily lose momentum. If we continue believing that momentum is a conserved quantity, we have to assume that a body losing momentum as it moves on the ground communicates its momentum to the ground (to the Earth).]

### 7.2.3 Relationship Between Momentum and Speed

There must certainly be a relationship between the speed and the momentum of a body: the faster it moves, the more momentum it has, and vice-versa. However, that does not make momentum and speed equivalent. The relation between the two depends upon the body.

If we let a glider move on a horizontal air track (where the air cushion insulates the glider from the track so it does not lose its momentum) and let it collide with an identical stationary glider in such a way that they couple and move together after the collision, we will find that the combined gliders move at half the speed of the first glider. Since the momentum of the first glider is divided up among two equal gliders, this observation tells us that momentum and speed are proportional:

$$p \sim v \tag{7.1}$$

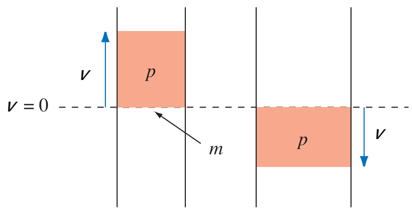



**Figure 7.5:** Fluid image of momentum. Momentum and speed can be positive or negative, just as electric charge. The cross section of the symbolic storage tank for momentum corresponds to the inertial mass of the body.

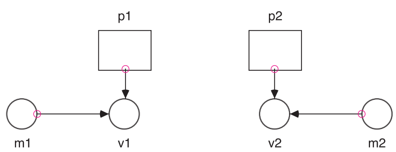
The factor of proportionality depends upon the body. We have every-day experience with the meaning of this factor: the larger this factor, the harder it is to set a body in motion. Therefore, this factor measures what we call the *inertia* of the body and we measure the inertia by the *inertial mass*:

$$p = mv \tag{7.2}$$

The inertial mass of a body is strictly proportional to its gravitational mass. For this reason, we can measure the inertial mass of a glider in terms of its weight.

### 7.2.4 A first dynamical model of the collision of two gliders

If momentum can be understood as the fluidlike quantity of translational motion, it will be represented by a reservoir in a system dynamics model (Fig. 7.6). Each of the gliders obtains its reservoir for its respective momentum. The speed—as the level of motion—is calculated from the momentum with the help of the mass. Taking analogous models in hydraulics for comparison, we see that the inertial mass of a body is the momentum capacitance (remember that the voltage of a capacitor is calculated from its charge just like speed is calculated from momentum).



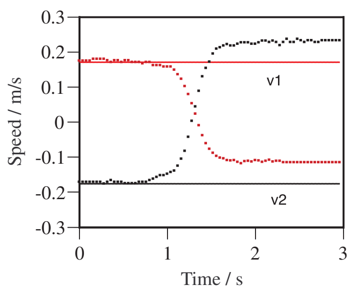




**Figure 7.6:** First model of the motion of the gliders. Each glider has a certain amount of momentum (represented by the reservoirs in the system dynamic model diagram). In this model, there are no interactions that could lead to transfer of momentum. As a result, momentum and speed stay constant.

Since there are no flows included with the reservoirs of momentum, the momenta of the two bodies must remain constant. Therefore, the speeds of the bodies will be constant in this model as well. In the graph on the right of Fig. 7.6, measured and simulated speeds are shown. The speeds calculated in the model are equal to the measured speeds at the beginning of the observation. As we can see, they remain constant throughout the simulation period.

What we learn here is this: *If there is no mechanical interaction, the momentum of a body remains constant*.

## 7.3 MOMENTUM TRANSFER IN TRANSLATIONAL PROCESSES

Hydraulic, electric or thermal processes are explained in terms of the transfer (and possibly the production) of the fluidlike quantities which are represented by reservoirs in system dynamic models. We will transfer this way of conceptualizing physical processes to phenomena of translational motion. In other words, we assume mechanical interactions to be represented by the transfer of momentum into and out of the bodies participating in the interactions.

### 7.3.1 Bodies gaining and losing momentum

A body can lose or gain momentum when it interacts mechanically with its surroundings—other bodies are part of the surroundings. A glider in our experiment can interact with the track, with the surrounding air, and with the other glider as a consequence of the interaction of the magnet.

If we assume that all mechanical interactions except for the one between the magnets of the gliders are negligible, all we have is the possibility of momentum transfer from one of the gliders to the other. If we assume the momentum leaving Glider 1 to arrive at Glider 2, there will be a flow of momentum between the two (see the representation of the flow between the two reservoirs in the system dynamics diagram of Fig. 7.7).

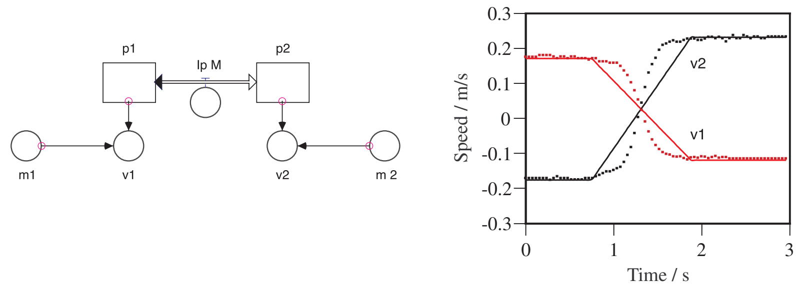



**Figure 7.7:** Model of the motion of the gliders with an interaction of constant strength during the period lasting from about 0.75 s to 1.85 s. During this period, momentum and speed change at a constant rate leading to a result that is only partially satisfactory. Note that initial and final speeds are calculated correctly. The total momentum of the gliders is conserved.

The speeds measured in the experiment indicate that the mechanical interaction lasts from about 0.75 s to a little later than 1.85 s. Outside this period of time, the speed and therefore the momenta of the bodies are (nearly) constant which tells us that momentum transfers must be negligible.

The important question remaining is this: what form does the momentum flow do

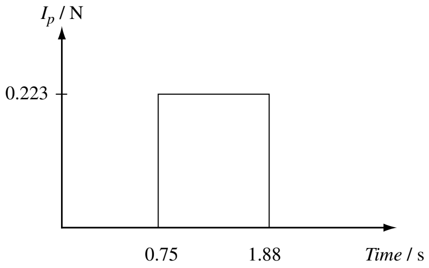



**Figure 7.8:** Model of the interaction of the magnets used in the model of Fig. 7.7.

to the interaction of the magnets take? Let us start with the simplest possible assumption of a constant flow during the time when the speeds and the momenta change; outside this period of time, the flow is assumed to be equal to zero:

Ip_M = IF (TIME > 0.75 AND TIME < 1.88) THEN 0.223 ELSE 0

The current of momentum between the gliders has been called $I_{pM}$. It is calculated according to the rule given above. The parameters in this relation (the times defining the period of interaction and the strength of the current) have been determined so that the simulated speeds approach the measured ones as closely as possible (see the diagram on the right of Fig. 7.7).

The comparison of data and simulation results shows that we definitely did not match the details of the interaction of the gliders. However, a couple of things are clearly working out: the final speeds measured in the experiment have been duplicated by the model which means that we have the correct momenta of the gliders before and after the collision. Moreover, since the only mechanical process is the momentum transfer from Glider 1 to Glider 2, we can conclude that the total momentum of the gliders is constant in the model (and in nature). We learn from our investigation, that the *momentum lost by Glider 1 is gained by Glider 2*. This is a special result of a general law of balance of momentum of a body:

$$\frac{dp}{dt} = I_{p1} + I_{p2} + \ldots \tag{7.3}$$

In our case, with two bodies exchanging momentum but otherwise not gaining or losing any, we have

$$\begin{aligned} \frac{dp_1}{dt} &= -I_{pM} \\ \frac{dp_2}{dt} &= I_{pM} \end{aligned} \tag{7.4}$$

There is an interesting alternative formulation of the model we have produced (see Fig. 7.7). Instead of having a single direct flow between the two reservoirs representing the exchange of momentum between the gliders, we give each reservoir a flow. This means that momentum is flowing out of (or into) Glider 1 and into (or out of) Glider 2.

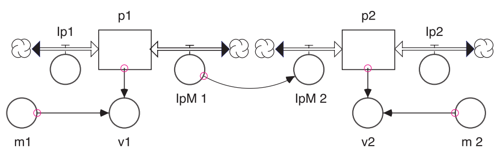



**Figure 7.9:** Alternative form of the model in Fig. 7.7: the interaction is represented by two flows, one for each body. Additional momentum currents are shown that might be used to model friction or other mechanical processes. Here, they are set equal to zero.

Since we assume the momentum leaving the first body to arrive at the second body, the two flows must be equal except for their signs:

$$I_{pM,1} = -I_{pM,2} \tag{7.5}$$

This is an example of a relation which is commonly called the principle of *action and reaction* in mechanics but clearly it can be applied to other processes as well.

## 7.4 Finding the Positions of the Bodies

The next step of model building concerns the computation of positions of the gliders from the speeds obtained. On the one hand, we are simply interested in this quantity since it is observed most easily. On the other hand, we will see that we want to make use of the information contained in the positions to create a better model of the interaction of the magnets (Section 7.5).

The calculation of positions from speeds involves the reverse operation of the one that leads to speeds calculated from positions: we have to integrate the speed of a body over time by simultaneously specifying the initial position. In system dynamics software, this is achieved by representing the speed as a flow and the position as a reservoir connected to this flow (see Fig. 7.10).

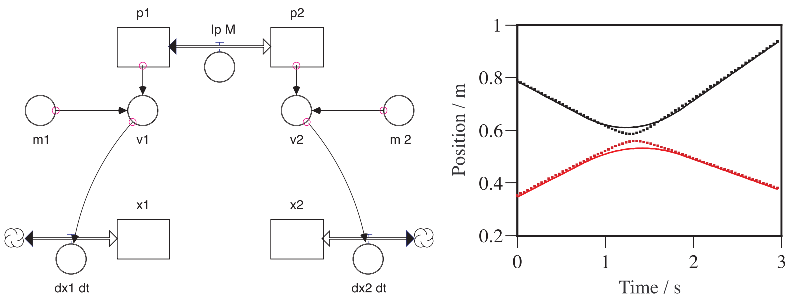



**Figure 7.10:** Calculating positions from speeds. In system dynamics software, the combination of a single flow with a stock represents the integration of the flow over time. Simulated and measured positions are shown in the diagram on the right.

## 7.5 A Model of the Interaction of the Magnets

The model constructed so far is not all that bad. It seems that it catches important elements of a description of the collision of two gliders. The missing link apparently is an adequate model for the flow of momentum between two magnets as they approach each other.

Every-day experience tells us that the repulsion and therefore the strength of the interaction increase when the ends of the magnets come closer. On the other hand, the observations of positions and speeds of the two gliders indicate that the changes of speed—and therefore of momenta—happen at a faster rate when the gliders are closer together. In summary, it makes sense to assume that the momentum current between the gliders introduced in the model in Fig. 7.7 or Fig. 7.10 measures the strength of the interaction and therefore should be larger the smaller the distance between the magnets. A possible mathematical relation that captures this feature is a momentum current which is inversely proportional to a power of the distance between the magnets:

$$I_{pM} = k \frac{1}{\Delta x^n} \tag{7.6}$$

An important detail of this model concerns the question of how to define the distance between the magnets since they have and extension in the direction of motion. Two possibilities come up spontaneously: either we take the distance between the ends of the magnets facing each other, or we take the distance between the centers of the magnets. Either way, we now have three parameters which have to be determined if we want to complete the model (Fig. 7.11).

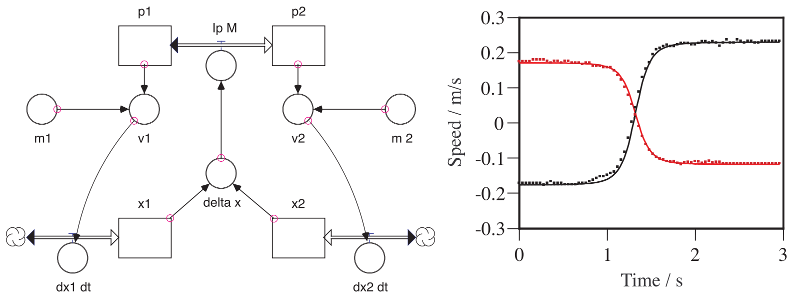



**Figure 7.11:** In the completed model, the flow of momentum from one glider to the other is made dependent upon the distance of the magnets. Note that we get very good agreement between data and simulation results for a particular choice of the form of the interaction. Compare to Fig. 7.7.

If we use the centers of the two magnets to define their distance, we only have two parameters left: $k$ and $n$ in the relation formulated in Equ. 7.6. It turns out that we get data and simulation results to agree very well for $n = 5$ and $k = 1.65 \cdot 10^{-5}$ SI units. The exponent $n$ determines the shape of the interaction-distance relation while $k$ measures the intrinsic strength of the magnets used in the experiment. With these parameters, the momentum current as a function of time or as a function of the distance of the centers of magnets are shown in Fig. 7.12.

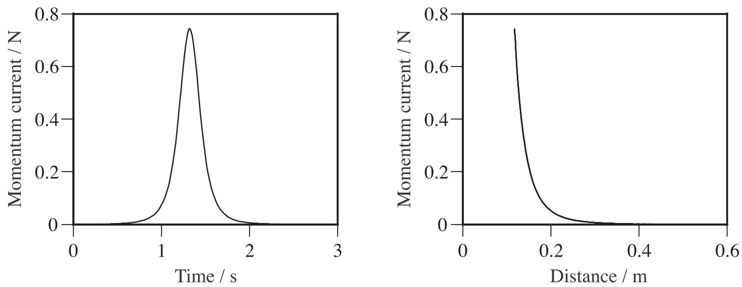



**Figure 7.12:** The momentum current as calculated in the model of Fig. 7.11 and Equ. 7.6 shown as functions of time and of the distance between the magnets.