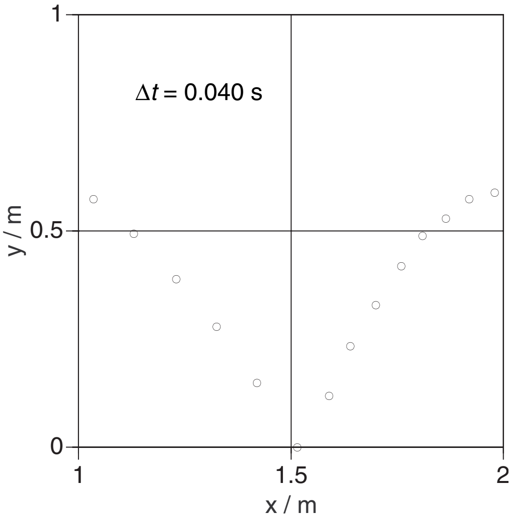
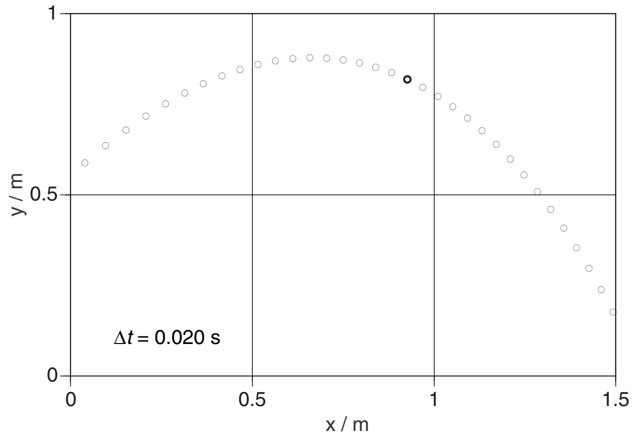
Physics Exam

1. A ping pong ball flies from the left and bounces off the floor (see graphic). The ball has a mass of 3.5 g. The impact lasts 0.0050 s. Ignore any possible rotational movement of the ball.
   a. Discuss the type(s) of interaction between the ball and the floor.
   b. What are the average values of the momentum currents which flow between the floor and the ball during impact?
   c. How much energy is dissipated during impact?



2. In the diagram included below, one sees the movement of a ball which is thrown through the air. Its mass is 3.62 g. Its radius is 3.8 cm. The air density is 1.2 kg/m^3.
   a. Determine the ball's acceleration at the point given (dark circle just after the highest point).
   b. How great is the force of the air upon the ball at this point?
   c. In what way does the air affect the ball?
   d. (Additional problem) Determine the air resistance without using the direct formula (remember, you do not know the air drag coefficient).



3. Jupiter needs 12 years to orbit once around the sun. Its moon Ganymede needs 7.155 days to orbit Jupiter.
   a. What is the orbital radius of Jupiter? (Assume that Jupiter's orbit is a circle. The distance of the Earth to the Sun is $150 \cdot 10^6$ km).
   b. Seen from the Earth, Ganymede moves away from the center of Jupiter a maximum of 5.76 arc minutes. What is the radius of Ganymede's orbital path?
   c. Use the data about Ganymede to determine the mass of Jupiter.

   Remark: In the Internet (http://hou.lbl.gov/studentreports/williamjup/jupiter.html) you can find a very interesting piece of work by a high school student concerning the determination of Jupiter's mass from observations of its moon Io.

4. An automobile drives with linearly increasing speed along a circular, horizontal street. Within 10 s the speed changes from an initial 36 km/h to 108 km/h. The street's radius of curvature is 200 m. The car's cross section for air resistance is 3.0 m^2. Air density and drag coefficient are 1.2 kg/m^3 and 0.5, respectively. The mass of the car is 1000 kg.
   a. How far does the car travel in these 10 seconds?
   b. Determine the acceleration vector of the car at 5.0 s after the start.
   c. Exactly at 10 s the street ends in a flat surface of ice, upon which there is no friction. Determine the car's movement upon the surface (direction of motion and initial acceleration).

SOLUTIONS

1. A ping pong ball flies from the left and bounces off the floor (see graphic). The ball has a mass of 3.5 g. The impact lasts 0.0050 s. Ignore any possible rotational movement of the ball.
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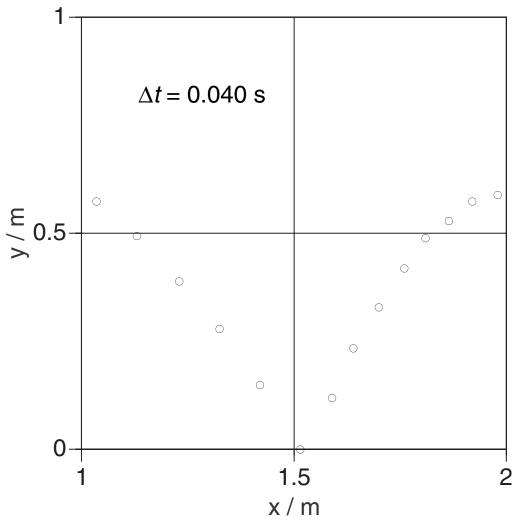
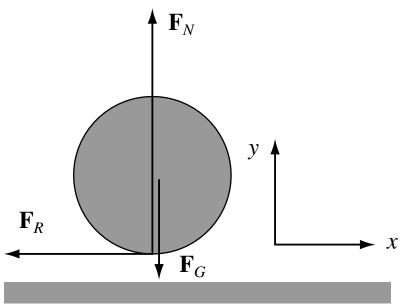


SOLUTION

a. Possibilities are compression and shearing. Compression leads to the normal force on the ball, shearing to dynamical friction:



b. Integrated form of Law of balance:

$$\Delta p_x = -I_{p,friction}\Delta t$$
$$\Delta p_y = -F_G \Delta t + I_{p,compression}\Delta t$$

Change of momentum calculated from change of velocity:

$$\Delta p_x = m\left(v_{x2} - v_{x1}\right)$$
$$\Delta p_y = m\left(v_{y2} - v_{y1}\right)$$

Therefore:

$$\bar{I}_{p,friction} = -\frac{m\left(v_{x2} - v_{x1}\right)}{\Delta t}$$
$$\bar{I}_{p,compression} = \frac{m\left(v_{y2} - v_{y1}\right) + mg\Delta t}{\Delta t}$$

Change of velocity from graph:

$$v_{x2} - v_{x1} = 0.075\text{m}/0.040\text{s} - 0.095\text{m}/0.040\text{s} = -0.50\,\text{m/s}$$
$$v_{y2} - v_{y1} = 0.12\text{m}/0.040\text{s} - \left(-0.15\text{m}/0.040\text{s}\right) = 6.75\,\text{m/s}$$

[Because of the effect of falling, the latter number is underestimated (by about 0.4 m/s).] Therefore:

$$\bar{I}_{p,friction} = -0.35\text{N}$$
$$\bar{I}_{p,compression} = 4.76\text{N}$$

[The latter value should be closer to 5 N.]

c. Energy dissipated equals kinetic energy lost:

$$W_{diss} = 0.5m\left(v_1^2 - v_2^2\right) = 0.5m\left(v_{x1}^2 + v_{y1}^2 - \left(v_{x2}^2 + v_{y2}^2\right)\right)$$
$$= 0.5 \cdot 0.0035 \cdot \left(2.375^2 + 3.75^2 - \left(1.875^2 + 3.0^2\right)\right)\text{J}$$
$$= 1.26 \cdot 10^{-2}\,\text{J}$$

2. In the diagram included below, one sees the movement of a ball which is thrown through the air. Its mass is 3.62 g. Its radius is 3.8 cm. The air density is 1.2 kg/m^3.
   a. Determine the ball's acceleration at the point given (dark circle just after the highest point).
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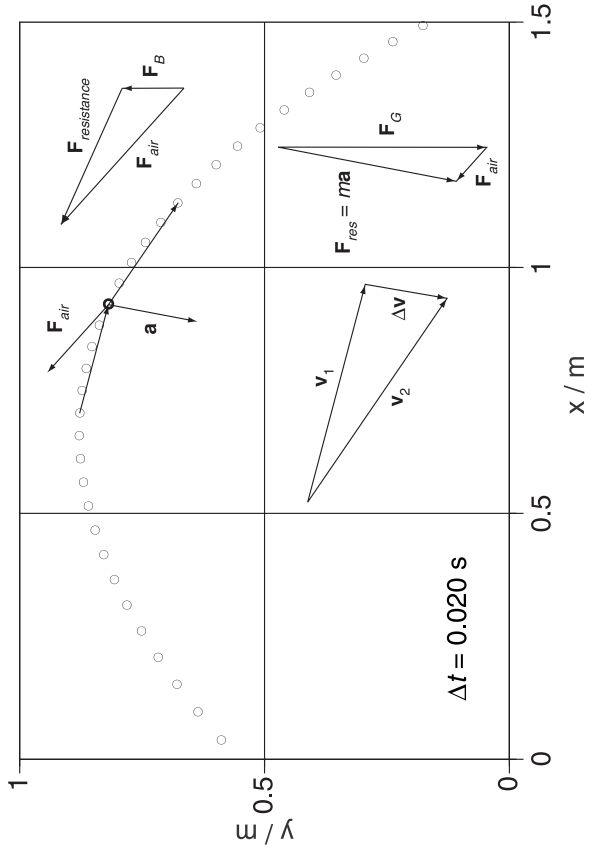
SOLUTION

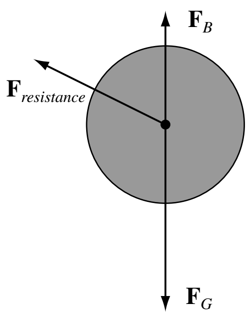
a. b. Graphical solution



In the following, magnitudes of the vectors constructed are given.
Velocities from changes of position: v1 = 2.28 m/s, v2 = 2.51 m/s, delta_v = 0.853 m/s (time interval: 0.10 s).

acceleration = 8.53 m/s2
F_res = ma = 0.031 N
F_air = 0.00813 N

c. The air affects the ball in two ways: by friction (air resistance) and buoyancy.



d. Additional problem: F_B = rho·g·volume = 0.00266 N (vertically up). Therefore:

F_resistance = 0.00665 N (F_x_resistance = – 0.00608 N, F_y_resistance = 0.00269 N).

3. Jupiter needs 12 years to orbit once around the sun. Its moon Ganymede needs 7.155 days to orbit Jupiter.
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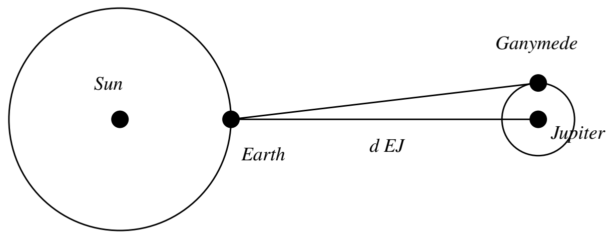
SOLUTION

a. From Kepler's third law:

$$r = \left(12^2\right)^{1/3} \text{AU} = 5.24\,\text{AU} = 7.86 \cdot 10^{11}\,\text{m}$$

b. Geometrical arrangement:



r_Ganymede = tan(phi)·d_EJ = tan(5.76/60°)·(5.24 – 1)· $1.5 \cdot 10^{11}$ m = $1.066 \cdot 10^9$ m

c. Motion of Ganymede:

$$m_G a_{n,Ganymede} = F_{G,Ganymede}$$
$$F_{G,Ganymede} = G\frac{m_J m_G}{r^2}$$
$$a_{n,Ganymede} = r\omega^2 = r\frac{4\pi^2}{T^2}$$

Therefore:

$$m_J = \frac{1}{G} r^3 \frac{4\pi^2}{T^2} = 1.87 \cdot 10^{27}\,\text{kg}$$

4. An automobile drives with linearly increasing speed along a circular, horizontal street. Within 10 s the speed changes from an initial 36 km/h to 108 km/h. The street's radius of curvature is 200 m. The car's cross section for air resistance is 3.0 m^2. Air density and drag coefficient are 1.2 kg/m^3 and 0.5, respectively. The mass of the car is 1000 kg.
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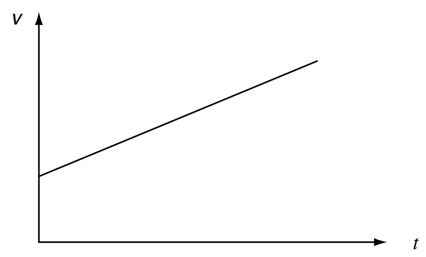
SOLUTION

a.



With constantly increasing speed, the tangential component of accelration is constant:

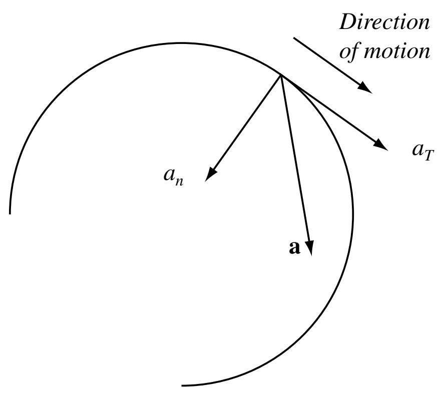
a_T = (v2 – v1)/delta_t = (30 m/s – 10 m/s)/10 s = 2.0 m/s^2

Distance travelled along path from integrating the speed:

s = 0.5·a_T·delta_t^2 + v_0·delta_t = 0.5·2.0·100 m + 10·10 m = 200 m

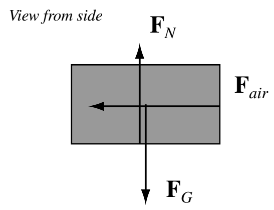
b. a_T has already been determined. a_n = v^2/R is obtained from the speed at t = 5 s:

a_n = (10 m/s + 2.0·5 m/s)^2 / 200 m = 2.0 m/s



a = 2.83 m/s in the direction indicated in the picture.

c. Upon the surface of ice, the following forces acting on the car can be identified:



vertical direction: weight and normal force

horizontal direction: air resistance opposed to direction of motion.

Therefore, the car must move—and continue to move—in a straight line. The direction is given by the direction of motion when the car enters the surface of ice. It's speed decreases because of air resistance. Right at the beginning the force of air resistance is

F_resistance = 0.5·0.5·3.0·1.2·30^2 N = 810 N

m·a_horizontal = – 810 N, therefore a = – 0.81 m/s^2