Physics Exam

1. A typical flat battery is connected to various load resistors and the voltage across the battery (terminal voltage) is measured (see Table 1).

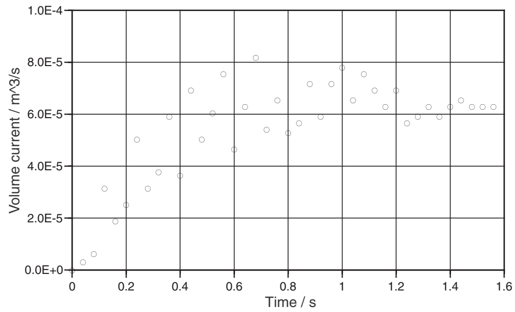
Table 1: Measurements

U_B / V	R_{load} / Ω
1.26	0.505
2.25	1.33
3.69	8.20
3.99	16.2
4.19	32.0

   a. Using the measurements, determine the current-terminal voltage-characteristic (I_Q-U_B diagram). Draw a well fitting straight line through the points.
   b. Determine the functional relation of the approximate linear function. What is the open circuit voltage?
   c. What is the internal resistance of the battery?
   d. Maximum Power Point. For which terminal voltage does the electrical power in the load resistor reach the maximum value? (Hint: Solve the problem graphically.)

2. A small electric pump is connected to a real battery with a (constant) internal resistance of 1.2 Ω. There is a resistor in the circuit with a (constant) resistance of 2.0 Ω. The open circuit voltage of the battery is 4.4 V. In steady-state, a volume current of $1.0 \cdot 10^{-4}$ m^3/s is measured as well as a pressure difference of $1.0 \cdot 10^4$ Pa (across the pump). The voltage across the load resistor is 2.02 V.
   a. What is the electric current through the circuit?
   b. What is the voltage across the pump?
   c. Determine the hydraulic power of the pump.
   d. What is the efficiency of the pump?

3. Water begins to flow in a thin horizontal hose. (The hose is the outflow at the bottom of a water tank.) In the diagram, measured data are given for the volume flow as a function of time. The hose has a length of 1.06 m and a radius of 4.0 mm. At the beginning, the water in the tank is 0.43 m high.



   a. Draw the simplest and smoothest curve through the data points which represents the expected flow of the water. (Use the graphic included on the reverse side). Determine the rate of change of the flow graphically at the point 0.40 s.
   b. What is the inductance of the water in the pipe and the inductive pressure difference at 0.40 s?
   c. Toward the end of the measured period, what is the pressure drop from the bottom of the tank to the inlet of the pipe (Bernoulli effect)?
   d. The flow is turbulent and the law of flow is represented by $\Delta p_R = k \cdot I_V^2$. Determine the resistance factor k with the help of the data.

4. Two small containers, each with a diameter of 10 cm, are filled with oil with a density of 1000 kg/m^3 and a viscosity of 0.080 Pa·s. Both containers are standing upon a table. A pipe connects the bottoms of the containers. It has a length of 0.50 m and a radius of 3.0 mm. Initially the oil stands 0.30 m high in the first container, and 0.10 m in the other.
   a. What is the (capacitive) time constant of the system?
   b. What are the rates of change of oil levels right at the beginning?
   c. Sketch the oil levels as a functions of time as precisely as possible (using numerical information where necessary and possible).

SOLUTIONS

1. A typical flat battery is connected to various load resistors and the voltage across the battery (terminal voltage) is measured (see Table 1).

Table 2: Measurements

U_B / V	R_{load} / Ω
1.26	0.505
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3.99	16.2
4.19	32.0

a. Using the measurements, determine the current-terminal voltage-characteristic (I_Q-U_B diagram). Draw a well fitting straight line through the points.
b. Determine the functional relation of the approximate linear function. What is the open circuit voltage?
c. What is the internal resistance of the battery?
d. Maximum Power Point. For which terminal voltage does the electrical power in the load resistor reach the maximum value? (Hint: Solve the problem graphically.)

SOLUTION

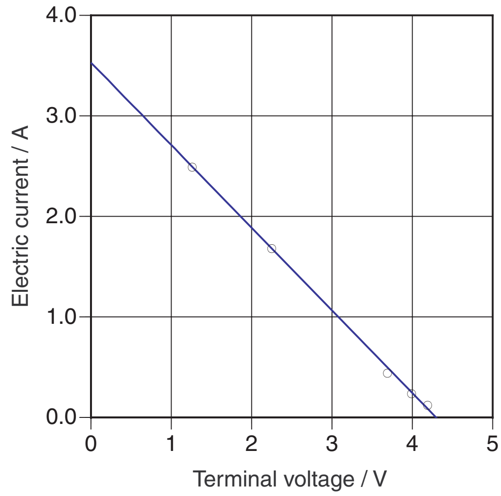
a. Calculate the electric current through the load resistor and the battery:

$$I_Q = \frac{U_B}{R_{load}}$$

(the terminal voltage U_B equals the voltage across the load resistor). Then prepare an I_Q-U_B diagram and fit a linear function through the data points.

Table 3: Measurements

U_B / V	R_{load} / Ω	I_Q / A
1.26	0.505	2.50
2.25	1.33	1.69
3.69	8.20	0.45
3.99	16.2	0.246
4.19	32.0	0.131



b. Linear function $I_Q(U_B)$:

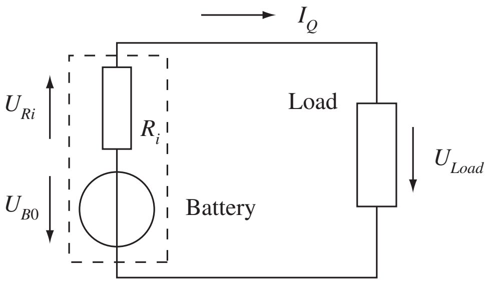
$$I_Q = -0.822\frac{\text{A}}{\text{V}}U_B + 3.53\text{A}$$

Therefore:

$$0 = -0.822\frac{\text{A}}{\text{V}}U_{B0} + 3.53\text{A}$$

$$U_{B0} = 3.53/0.822\text{V} = 4.29\text{V}$$

c.



Loop rule:

$$U_{B0} = U_{Ri} + U_{Load} \quad (U_{Load} = U_B)$$

Resistance law:

$$U_{Ri} = R_i I_Q$$

Therefore

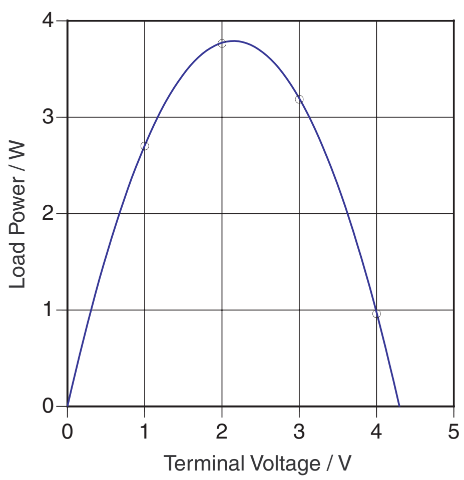
$$R_i I_Q = U_{B0} - U_B$$
$$R_i\left(-0.822\frac{\text{A}}{\text{V}}U_B + 3.53\text{A}\right) = 3.53/0.822\text{V} - U_B$$
$$R_i = 1.22\Omega$$

(You can choose any value for U_B to get the value of R_i.)

d. Calculate the power as a function of UB and represent it in a diagram. Find the maximum value.

Table 4: Measurements

U_B / V	I_Q / A	$\mathcal{P}_{el}$ / W
1	2.708	2.708
2	1.886	3.772
3	1.064	3.192
4	0.242	0.968



Maximum Power at about 2.15 V terminal power. The load resistance at this point is 1.22 W.

Formal solution:

$$\mathcal{P} = U_B I_Q = U_B\left(-0.822\frac{\text{A}}{\text{V}}U_B + 3.53\text{A}\right)$$
$$= -0.822\frac{\text{A}}{\text{V}}U_B^2 + 3.53\text{A} \cdot U_B$$

$$\frac{d\mathcal{P}}{dU_B} = -2 \cdot 0.822\frac{\text{A}}{\text{V}}U_B + 3.53\text{A}$$
$$\frac{d\mathcal{P}}{dU_B} = 0 \quad \Rightarrow \quad U_{B,\max} = 2.15\text{V}$$

2. A small electric pump is connected to a real battery with a (constant) internal resistance of 1.2 Ω. There is a resistor in the circuit with a (constant) resistance of 2.0 Ω. The open circuit voltage of the battery is 4.4 V. In steady-state, a volume current of $1.0 \cdot 10^{-4}$ m^3/s is measured as well as a pressure difference of $1.0 \cdot 10^4$ Pa (across the pump). The voltage across the load resistor is 2.02 V.
   a. What is the electric current through the circuit?
   b. What is the voltage across the pump?
   c. Determine the hydraulic power of the pump.
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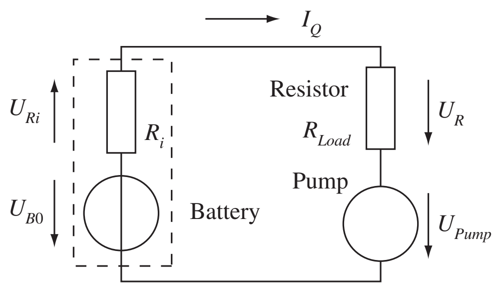
SOLUTION

a. Circuit diagram:



$$I_Q = \frac{U_R}{R_{Load}} = \frac{2.02}{2.0}\text{A} = 1.01\text{A}$$

b.

$$U_{Pump} = U_{B0} - (R_{load} + R_i)I_Q$$
$$= 4.4\text{V} - 3.2 \cdot 1.01\text{V} = 1.17\text{V}$$

c.

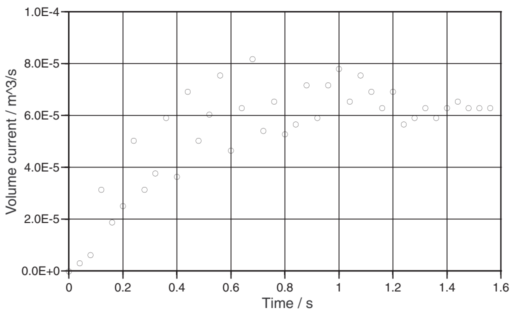
$$\mathcal{P}_{hydr} = \Delta p_{pump} I_V = 10^4 \cdot 10^{-4}\,\mathrm{W} = 1.0\,\mathrm{W}$$

d.

$$\eta = \mathcal{P}_{hydr} / \mathcal{P}_{el,\,pump}$$

$$\eta = \mathcal{P}_{hydr} / \left(U_{Pump} I_Q\right) = 1.0/(1.17 \cdot 1.01) = 0.846$$

3. Water begins to flow in a thin horizontal hose. (The hose is the outflow at the bottom of a water tank.) In the diagram, measured data are given for the volume flow as a function of time. The hose has a length of 1.06 m and a radius of 4.0 mm. At the beginning, the water in the tank is 0.43 m high.



a. Draw the simplest and smoothest curve through the data points which represents the expected flow of the water. (Use the graphic included on the reverse side). Determine the rate of change of the flow graphically at the point 0.40 s.
b. What is the inductance of the water in the pipe and the inductive pressure difference at 0.40 s?
c. Toward the end of the measured period, what is the pressure drop from the bottom of the tank to the inlet of the pipe (Bernoulli effect)?
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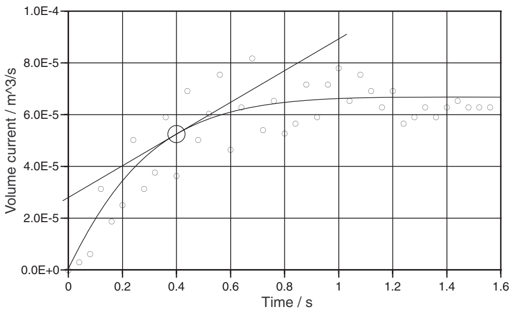
SOLUTION

a.



$dI_V/dt \approx 6.11 \cdot 10^{-5}\ \mathrm{m^3/s}$ / 1.0 s = $6.11 \cdot 10^{-5}\ \mathrm{m^3/s^2}$.

b.

$$L_V = \frac{\rho l}{\pi r^2} = \frac{1000 \cdot 1.06}{\pi \cdot 0.0040^2} = 2.11 \cdot 10^7 \frac{\mathrm{kg}}{\mathrm{m^4}}$$

$$\Delta p_L = -L_V \frac{dI_V}{dt} = -2.11 \cdot 10^7 \cdot 6.11 \cdot 10^{-5}\,\mathrm{Pa} = -1290\,\mathrm{Pa}$$

c. Determine the speed of the flow from the current at the end. The current is read from the graph:

$$\Delta p_B = -\frac{1}{2} \rho v^2$$

$$v = I_V / \left(\pi r^2\right)$$

$$= 6.68 \cdot 10^{-5} / \left(\pi \cdot 0.0040^2\right)\,\mathrm{m/s} = 1.33\,\mathrm{m/s}$$

$$\Delta p_B = -0.5 \cdot 1000 \cdot 1.33^2\,\mathrm{Pa} = -883\,\mathrm{Pa}$$

d. At the end (constant current), the inductive pressure difference is zero. Therfore, the resistive pressure difference across the pipe is equal to the pressure difference across the fluid column in the tank and the Bernoulli effect:

$$\Delta p_R = \rho g h - \Delta p_B$$

$$= 1000 \cdot 9.81 \cdot 0.43\,\mathrm{Pa} - 883\,\mathrm{Pa} = 3340\,\mathrm{Pa}$$

$$k = \frac{\Delta p_R}{I_V^2} = \frac{3340\,\mathrm{Pa}}{\left(6.68 \cdot 10^{-5}\right)^2} \frac{\mathrm{Pa \cdot s^2}}{\mathrm{m^6}} = 7.47 \cdot 10^{11} \frac{\mathrm{Pa \cdot s^2}}{\mathrm{m^6}}$$

4. Two small containers, each with a diameter of 10 cm, are filled with oil with a density of 1000 kg/m^3 and a viscosity of 0.080 Pa·s. Both containers are standing upon a table. A pipe connects the bottoms of the containers. It has a length of 0.50 m and a radius of 3.0 mm. Initially the oil stands 0.30 m high in the first container, and 0.10 m in the other.
a. What is the (capacitive) time constant of the system?
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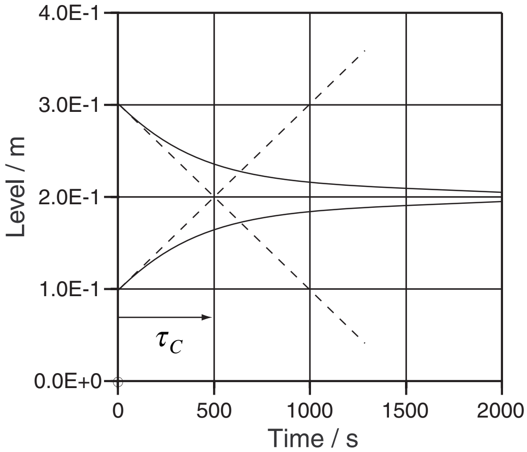

SOLUTION

a.

$$K_V = \frac{1}{2}\frac{\pi r_{tank}^2}{\rho g} = \frac{\pi \cdot 0.050^2}{2 \cdot 1000 \cdot 9.81}\frac{\mathrm{m}^3}{\mathrm{Pa}} = 4.00 \cdot 10^{-7}\frac{\mathrm{m}^3}{\mathrm{Pa}}$$

$$R_V = \frac{8\eta l}{\pi r_{pipe}^4} = \frac{8 \cdot 0.080 \cdot 0.50}{\pi \cdot 0.0030^4}\frac{\mathrm{Pa \cdot s}}{\mathrm{m}^3} = 1.26 \cdot 10^9 \frac{\mathrm{Pa \cdot s}}{\mathrm{m}^3}$$

$$\tau_C = R_V K_V = 503\mathrm{s}$$

b. The rate of change of height is found from the rate of change of volume, which in turn is found from the (sum of) the volume currents. The latter is found from the resistive pressure difference which must be equal to the capacitive pressure difference across the tanks (no induction, no Bernoulli effect).

$$\dot{V}_1 = -|I_V| \quad , \quad \dot{V}_2 = -|I_V|$$

$$|I_V| = \left|\frac{\Delta p_R}{R_V}\right|$$

$$|\Delta p_R| = |\Delta p_C| \quad , \quad |\Delta p_C| = \rho g |\Delta h|$$

$$\dot{V}_1 = A_1 \dot{h}_1 \quad , \quad \dot{V}_2 = A_2 \dot{h}_2$$

Therefore

$$\dot{h}_1 = -\frac{1}{\pi r_{tank}^2}\dot{V}_1 = -\frac{1}{\pi r_{tank}^2}|I_V| = -\frac{1}{\pi r_{tank}^2}\left|\frac{\Delta p_R}{R_V}\right|$$

$$= -\frac{1}{\pi r_{tank}^2}\left|\frac{\Delta p_C}{R_V}\right| = -\frac{1}{\pi r_{tank}^2}\frac{\rho g |\Delta h|}{R_V} = -1.98 \cdot 10^{-4}\frac{\mathrm{m}}{\mathrm{s}}$$

and

$$\dot{h}_2 = 1.98 \cdot 10^{-4}\frac{\mathrm{m}}{\mathrm{s}}$$

c. The levels will meet at a value of 0.20 m. If they continued to change at the initial rate of 0.20 mm/s, the would be equal after 500 s (time constant). After 500 s, the level difference must have decreased to 36% of its initial value.