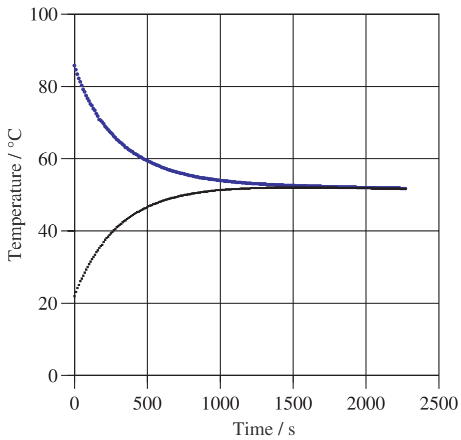
PHYSICS EXAM

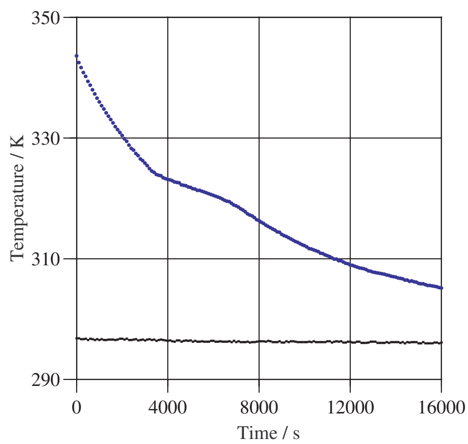
1. Where necessary, explain your answers to the following questions.
   a. Which properties of entropy appear in the law of balance of entropy?
   b. What is the dynamical form of the balance of entropy for water being continuously stirred in a perfectly insulated container? (Consider water and mixer as the system.)
   c. How do you determine the zero point of the Kelvin scale with the help of the gas thermometer?
   d. What is the *TS* diagram of the process of fusion (solidification) of a liquid? (Add an arrow for the direction of the process.)
   e. What is the specific heat – temperature diagram of a substance whose entropy capacitance is constant?
   f. How do you calculate the entropy production rate of an immersion heater, and which data do you take in the lab to be able to perform the calculation?
   g. Two equal amounts of a liquid having different initial temperatures are mixed in an insulated container. How can you make use of the heat capacitance – temperature diagram of the substance to determine the final temperature of the mixture? Is the result always equal to the average temperature of the initial values?
   h. A house is in an environment having a constant temperature of 0°C. If you turn the heating off, the temperature of the building decreases from 20°C to 7.4°C in 24 hours. The energy conductance for heat loss of the house is 300 W/K. What is the energy capacitance of the building?

2. Two amounts of water having equal mass of 0.951 kg each are inside two chambers separated by a thin metal wall. The dimensions of the chambers are 10 x 10 x 10 cm. The container chambers are well insulated to the environment.

   Hot water is filled into one of the chambers, cold water into the other. The water is mixed thoroughly all the time. The temperatures are measured as functions of time (see the enlargement of the diagram below).
   a. What is the convective heat transfer coefficient from the water to the metal wall? Note that the thermal resistance of the metal wall itself is very small. (Use the data of the experiment and neglect heat loss to the environment and the effect of the mixer).
   b. What is the change of the energy of each of the bodies of water during the first 500 s?
   c. Use the data to determine as carefully as possible the entropy conductance (or the energy conductance) for heat loss to the environment for the entire container. The power of the mixers in the two chambers adds up to 1.0 W, and the ambient temperature is equal to 22°C.



3. In a thin-walled metal container there are 200 g of wax. The container is immersed in 0.80 kg of water in a glass container which is very well insulated at the top and at the bottom. Water and wax are very hot initially. Now they are cooling in the environment (in the diagram, the temperatures of water and environment are shown; enlargement of the diagram see below). Between t_1 = 3500 s and t_2 = 7000 s, we observe a plateau in the water temperature.



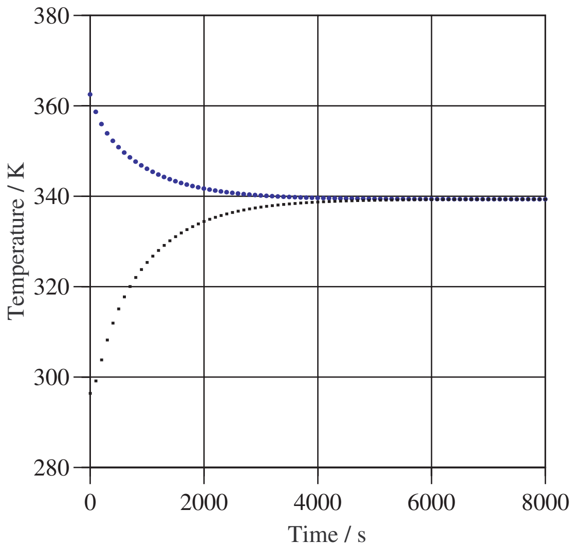
   a. Explain in words why there is a plateau in the water temperature.
   b. Estimate the melting temperature of the wax.
   c. For the center of the plateau, determine the rate of change of temperature and the rate of change of entropy of the water.

d. For the time corresponding to the center of the plateau, how large is the entropy current from the wax to the water? The entropy conductance from the water to the ambient air is $2.1 \cdot 10^{-3}$ W/K^2. (Neglect all entropy production rates due to entropy transport.)
e. Using the previous results, estimate how much entropy has flowed out of the wax during solidification. What is therefore the specific latent entropy of fusion of the wax?

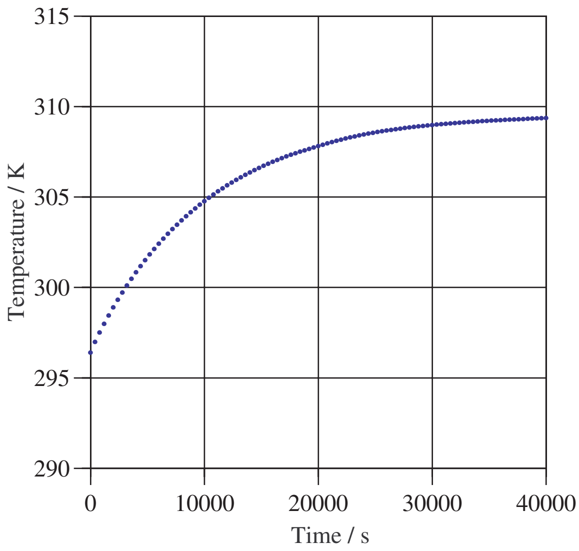
4. Two different experiments are performed with a thick-walled PVC container. They are used to determine the specific heat (specific energy capacitance) and the thermal conductivity (energy conductivity) of PVC. When you have found these values, also determine the specific entropy capacitance and the entropy conductivity.

   Data for the container: Inside radius: 3.0 cm, outer radius: 4.65 cm, inside height: 0.105 m, density: 1400 kg/m^3.

   Experiment 1: The container is completely insulated from the environment. The container has an initial temperature of 23.4°C. It is filled with hot water having a temperature of 89.5°C. Everything is now sealed, and the water temperature and the temperature at the middle of the container wall are measured (see graph).



   Experiment 2: Lid and bottom are perfectly insulated. Water and container have an initial temperature of 23.4°C, the same value as the (constant) temperature of the environment. The magnetic mixer dissipates energy at a rate of 2.0 W. Take values of 200 W/(K·m^2) and 13 W/(K·m^2) for the heat transfer coefficients on the inside and the outside of the PVC container. The graph shows the water temperature as a function of time.



SOLUTIONS

1. Where necessary, explain your answers to the following questions.
   a. Which properties of entropy appear in the law of balance of entropy?

      SOLUTION: Entropy can be stored, can flow, and can be produced.

   b. What is the dynamical form of the balance of entropy for water being continuously stirred in a perfectly insulated container? (Consider water and mixer as the system.)

      SOLUTION: No entropy flow, but entropy production and storage:

$$\dot{S} = \Pi_S$$

   c. How do you determine the zero point of the Kelvin scale with the help of the gas thermometer?

      SOLUTION: Measure the pressure of the gas in a container (at constant volume) as a function of the Celsius temperature. Interpolate the data with a linear function and extrapolate to a value of zero for the pressure.

   d. What is the *TS* diagram of the process of fusion (solidification) of a liquid? (Add an arrow for the direction of the process.)

      SOLUTION: Horizontal line to the left (constant temperature, decreasing entropy).

   e. What is the specific heat – temperature diagram of a substance whose entropy capacitance is constant?

      SOLUTION: $c(T) = T \cdot k\_s$, where k_s is a constant; therefore, $c(T)$ is proportional to the Kelvin temperature of the substance.

   f. How do you calculate the entropy production rate of an immersion heater, and which data do you take in the lab to be able to perform the calculation?

      SOLUTION: Measure voltage and electric current for the heater, and the temperature of the liquid it is immersed in. Then calculate

$$\Pi_S = \frac{U I_Q}{T}$$

   g. Two equal amounts of a liquid having different initial temperatures are mixed in an insulated container. How can you make use of the heat capacitance – temperature diagram of the substance to determine the final temperature of the mixture? Is the result always equal to the average temperature of the initial values?

      SOLUTION: Draw $c(T)$ in the diagram, and note the initial temperatures of the bodies of liquid (T_{i1} and T_{i2}). The areas under the $c(T)$ curve between T_{i1} and the final temperature T_f, and T_f and T_{i2}, respectively, have to be equal. This defines T_f.

   h. A house is in an environment having a constant temperature of 0°C. If you turn the heating off, the temperature of the building decreases from 20°C to 7.4°C in 24 hours. The energy conductance for heat loss of the house is 300 W/K. What is the energy capacitance of the building?

      SOLUTION: The home will eventually cool to 0°C. Typically, the cooling curve is a decaying exponential function, and 7.4°C is 1/e of the initial temperature difference of 20°C. Therefore, the time span of 24 h corresponds to the time constant of the process:

$$\tau = RC = \frac{1}{G}C \quad \Rightarrow \quad C = G\tau$$

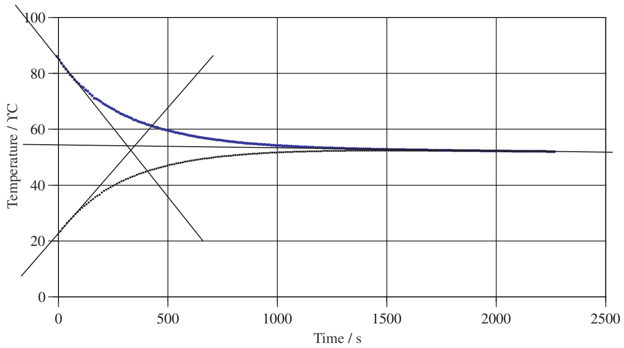
$$C = 300 \cdot 24 \cdot 3600 \frac{\text{J}}{\text{K}} = 2.59 \cdot 10^7 \frac{\text{J}}{\text{K}}$$

2. Two amounts of water having equal mass of 0.951 kg each are inside two chambers separated by a thin metal wall. The dimensions of the chambers are 10 x 10 x 10 cm. The container chambers are well insulated to the environment.

   Hot water is filled into one of the chambers, cold water into the other. The water is mixed thoroughly all the time. The temperatures are measured as functions of time (see the enlargement of the diagram below).

   a. What is the convective heat transfer coefficient from the water to the metal wall? Note that the thermal resistance of the metal wall itself is very small. (Use the data of the experiment and neglect heat loss to the environment and the effect of the mixer).

SOLUTION:



Determine the rate of change of temperature of one of the bodies of water (at $t = 0$ s). From that, determine the rate of change of energy of the body; this must be equal to the energy flow from the hot to the cold body (neglecting heat loss to the environment, and friction because of mixing):

$$\dot{T}_1 \approx -0.098\frac{K}{s}$$
$$\dot{W}_1 = C_1\dot{T}_1 = m_1c_1\dot{T}_1 = 0.951\cdot 4200\cdot(-0.098)\text{W} = -391\text{W}$$
$$I_{W,water\,to\,water} = 391\text{W}$$

Determine the temperature difference between the bodies of water (at $t = 0$ s). This determines the energy conductance and the total heat transfer coefficient (from water to water):

$$h = \frac{I_{W,water\,to\,water}}{A(T_1 - T_2)} = \frac{391}{0.10\cdot 0.10\cdot 63.7}\frac{\text{W}}{\text{K}\cdot\text{m}^2} = 614\frac{\text{W}}{\text{K}\cdot\text{m}^2}$$

Neglecting the resistance of the wall, the transfer coefficient from a body of water to the wall is about twice this value, i.e., it is 1200 W/(K·m^2).

b. What is the change of the energy of each of the bodies of water during the first 500 s?

SOLUTION: Determine the temperature change for each of the bodies of water in the first 500 s:

$$\Delta T_1 \approx -25.1\text{K}$$
$$\Delta T_2 \approx 25.1\text{K}$$

Therefore:

$$\Delta W_1 = m_1c_1\Delta T_1 = 0.951\cdot 4200\cdot(-25.1)\text{J} = -100\text{kJ}$$
$$\Delta W_2 = m_2c_2\Delta T_2 = 0.951\cdot 4200\cdot 25.1\text{J} = 100\text{kJ}$$

c. Use the data to determine as carefully as possible the entropy conductance (or the energy conductance) for heat loss to the environment for the entire container. The power of the mixers in the two chambers adds up to 1.0 W, and the ambient temperature is equal to 22°C.

SOLUTION: The common temperature of the two bodies of water (after about 1400 s) slowly decreases, indicating a loss of entropy to the environment. Taking entropy production into consideration, the balance of energy for both bodies of water is

$$\dot{W}_{1+2} = I_{W,mixer} - I_{W,loss}$$

This is equal to

$$\dot{W}_{1+2} = 2mc\dot{T}$$

Therefore

$$I_{W,loss} = I_{W,mixer} - 2mc\dot{T} = 1.0\text{W} - 2\cdot 0.951\cdot 4200\cdot\left(-1.04\cdot 10^{-3}\right)\text{W} = 9.3\text{W}$$

The rate of change of temperature of both bodies of water after about 1400 s was measured to be -1.04 mK/s. With a temperature difference between the bodies of water and the environment of about 30 K, the energy conductance for loss for the entire container is 0.31 W/K. Therefore, the entropy conductance for loss is about 0.40 W/K / (52 + 273) K = 0.96·10^{-3} W/K^2.

3. In a thin-walled metal container there are 200 g of wax. The container is immersed in 0.80 kg of water in a glass container which is very well insulated at the top and at the bottom. Water and wax are very hot initially. Now they are cooling in the environment (in the diagram, the temperatures of water and environment are shown; enlargement of the diagram see below). Between $t_1 = 3500$ s and $t_2 = 7000$ s, we observe a plateau in the water temperature.

a. Explain in words why there is a plateau in the water temperature.

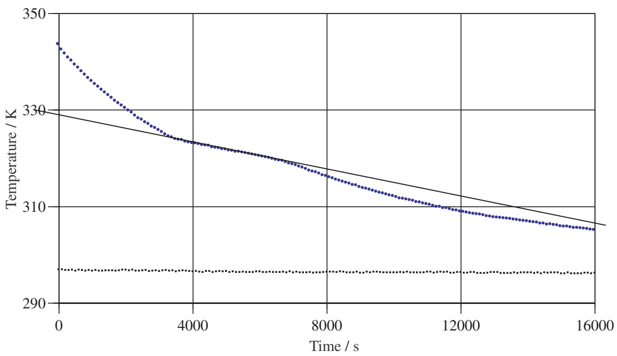
SOLUTION: During this time, the wax solidifies and give off entropy at a constant temperature. This somewhat stabilizes the temperature of the water.

b. Estimate the melting temperature of the wax.

SOLUTION: The phase change starts when the water temperature is about 324 K = 51°C. The temperature of the wax at this point is about the same, meaning that the temperature of phase change is about 52°C.

c. For the center of the plateau, determine the rate of change of temperature and the rate of change of entropy of the water.

SOLUTION:



$$\dot{T} \approx -1.4 \cdot 10^{-3}\ \frac{\mathrm{K}}{\mathrm{s}}$$

$$\dot{S} = mk_S\dot{T} = m\frac{c}{T}\dot{T}$$

$$= 0.80\frac{4200}{50+273}\left(-1.4 \cdot 10^{-3}\right)\frac{\mathrm{W}}{\mathrm{K}} = -1.46 \cdot 10^{-2}\ \frac{\mathrm{W}}{\mathrm{K}}$$

d. For the time corresponding to the center of the plateau, how large is the entropy current from the wax to the water? The entropy conductance from the water to the ambient air is $2.1 \cdot 10^{-3}$ W/K^2. (Neglect all entropy production rates due to entropy transport.)

SOLUTION: Use the law of balance of entropy of water:

$$\dot{S} = I_{S,from\ wax} - I_{S,to\ air}$$

The entropy loss to air is calculated from the temperature difference between water and air and the entropy conductance:

$$I_{S,to\ air} = G_S\left(T_w - T_{air}\right) = 2.1 \cdot 10^{-3} \cdot 25\frac{\mathrm{W}}{\mathrm{K}}$$

$$= 5.25 \cdot 10^{-2}\ \frac{\mathrm{W}}{\mathrm{K}}$$

Therefore, the entropy current from wax to water is equal to $3.8 \cdot 10^{-2}$ W/K.

e. Using the previous results, estimate how much entropy has flowed out of the wax during solidification. What is therefore the specific latent entropy of fusion of the wax?

SOLUTION: If the entropy current out of the wax were constant, the wax would have lost a total of 133 J/K of entropy (the real value is smaller since the entropy current is smaller than the assumed value at the beginning and at the end of the phase). Accepting the result, the specific latent entropy of the wax is 133/0.2 J/(K·kg) = 660 J/(K·kg). (A comparison with a detailed model shows that the value is overestimated by about 20%).

4. Two different experiments are performed with a thick-walled PVC container. They are used to determine the specific heat (specific energy capacitance) and the thermal conductivity (energy conductivity) of PVC. When you have found these values, also determine the specific entropy capacitance and the entropy conductivity.

   Data for the container: Inside radius: 3.0 cm, outer radius: 4.65 cm, inside height: 0.105 m, density: 1400 kg/m^3.

   Experiment 1: The container is completely insulated from the environment. The container has an initial temperature of 23.4°C. It is filled with hot water having a temperature of 89.5°C. Everything is now sealed, and the water temperature and the temperature at the middle of the container wall are measured (see graph).

   SOLUTION: Experiment 1 can be used to calculate the specific heat (energy capacitance) of PVC from the initial and final temperatures with the help of the balance of energy. Neglecting the lid and the bottom, the mass of the container is 0.583 kg, and the mass of the water filling the container is 0.297 kg. Therefore

$$m_w c_w\left(T_{wi} - T_f\right) = m_{PVC}c_{PVC}\left(T_f - T_{PVCi}\right)$$

$$c_{PVC} = \frac{m_w c_w\left(T_{wi} - T_f\right)}{m_{PVC}\left(T_f - T_{PVCi}\right)} =$$

$$= \frac{0.297 \cdot 4200 \cdot (363 - 339)}{0.583 \cdot (339 - 296)}\frac{\mathrm{J}}{\mathrm{K \cdot kg}}$$

$$= 1190\frac{\mathrm{J}}{\mathrm{K \cdot kg}}$$

   Experiment 2: Lid and bottom are perfectly insulated. Water and container have an initial temperature of 23.4°C, the same value as the (constant) temperature of the environment. The magnetic mixer dissipates energy at a rate of 2.0 W. Take values of 200 W/(K·m^2) and 13 W/(K·m^2) for the heat transfer coefficients on the inside and the outside of the PVC container. The graph shows the water temperature as a function of time.

SOLUTION: Experiment 2 can be used to determine the conductivity of the PVC. The balance of energy for the final steady-state (when the temperature of the water is not changing any longer) tell us that

$$\mathcal{P}_{diss} = I_{W,loss}$$

With the law of heat loss we have

$$\begin{aligned} I_{W,loss} &= \frac{1}{R_W}\left(T_w - T_{air}\right) \\ R_W &= \frac{T_w - T_{air}}{I_{W,loss}} = \frac{309-296}{2.0} = 6.5\frac{\mathrm{K}}{\mathrm{W}} \end{aligned}$$

and with the expression for the resistance we have

$$\begin{aligned} R_W &= \frac{1}{A_i h_i} + \frac{\Delta x}{A_m \lambda} + \frac{1}{A_o h_o} \\ \lambda &= \frac{\Delta x}{A_m}\left(R_W - \frac{1}{A_i h_i} - \frac{1}{A_o h_o}\right)^{-1} \\ &= \frac{0.0165}{0.0252}\left(6.5 - \frac{1}{0.0198\cdot 200} - \frac{1}{0.0307\cdot 13}\right)^{-1}\frac{\mathrm{W}}{\mathrm{K}\cdot\mathrm{m}} \\ &= 0.175\frac{\mathrm{W}}{\mathrm{K}\cdot\mathrm{m}} \end{aligned}$$