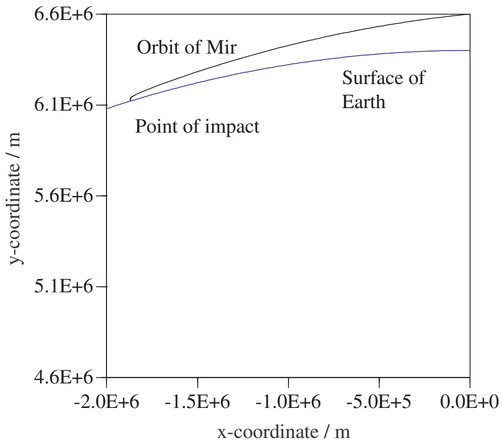
Physics Exam

1. The Mir space station's crash to earth is simulated with the help of a strongly simplified model (see figure). A greatly enlarged depiction of the crash is on an accompanying page. The mass of Mir is 60 tons. There is a strong decrease of air density when measured vertically upwards from the earth's surface.
   a. Determine the acceleration of Mir graphically at the given (gray) point shortly before impact.
   b. What is the air resistance at this point?
   c. Determine the rate at which energy is dissipated at this moment.

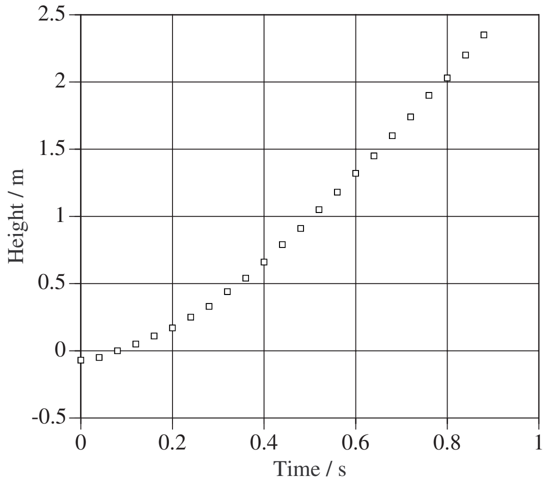


2. Muffin cups are dropped from a certain height, and the terminal (maximal) velocity is measured (see the entries in the table). The values are the results of a series of measurements with one, two, three… cups stacked into each other.

Table 1: Final velocity of falling cups

***n* cups**	***m* / kg**	**v_{max} / m/s**
1	2.94E-04	1.67
2	5.88E-04	2.40
3	8.82E-04	3.00
4	1.18E-03	3.35
5	1.47E-03	3.80

   a. What kind of force law is best for this case according to the data? Is the force of air resistance proportional to the velocity, or rather to the velocity squared? Or is it proportional to the third power of the velocity? Perform a (graphical) analysis based on the data.
   Derive the theoretical relation between terminal velocity and mass of the body for the force law which you extracted from the data.
   b. In the diagram below, the motion of the body consisting of 5 cups is shown. How much energy has been dissipated because of air resistance up to 0.80 s?



3. For the following questions, give answers with explanations or detailed reasoning.
   a. A ball performs the typical motion of a projectile. Neglect air resistance. Consider a point along the trajectory where the ball is moving downward. Is there a tangential component of the acceleration of the ball?
   b. Is it possible for the acceleration of a point moving on an elliptic orbit to be zero?
   c. A train moves along a horizontal curved track (not a circle!) while getting faster. The cars tilt inward. In the dining car, there is a glass on the table. It is slowly sliding outward (with respect to the curve of the tracks) toward the window. Draw all necessary free-body diagrams, i.e., identify all the forces acting on the glass. (Friction is possible.)
   d. What is the direction of the sum of all forces acting on the glass in the previous example?
   e. A disc having a mass of 0.75 kg is moving on an air table toward another disc at rest (mass: 0.50 kg). There is a collision. The speed of the first disk is 0.35 m/s before the collision, and 0.25 m/s afterwards. The second disk has a speed of 0.31 m/s after

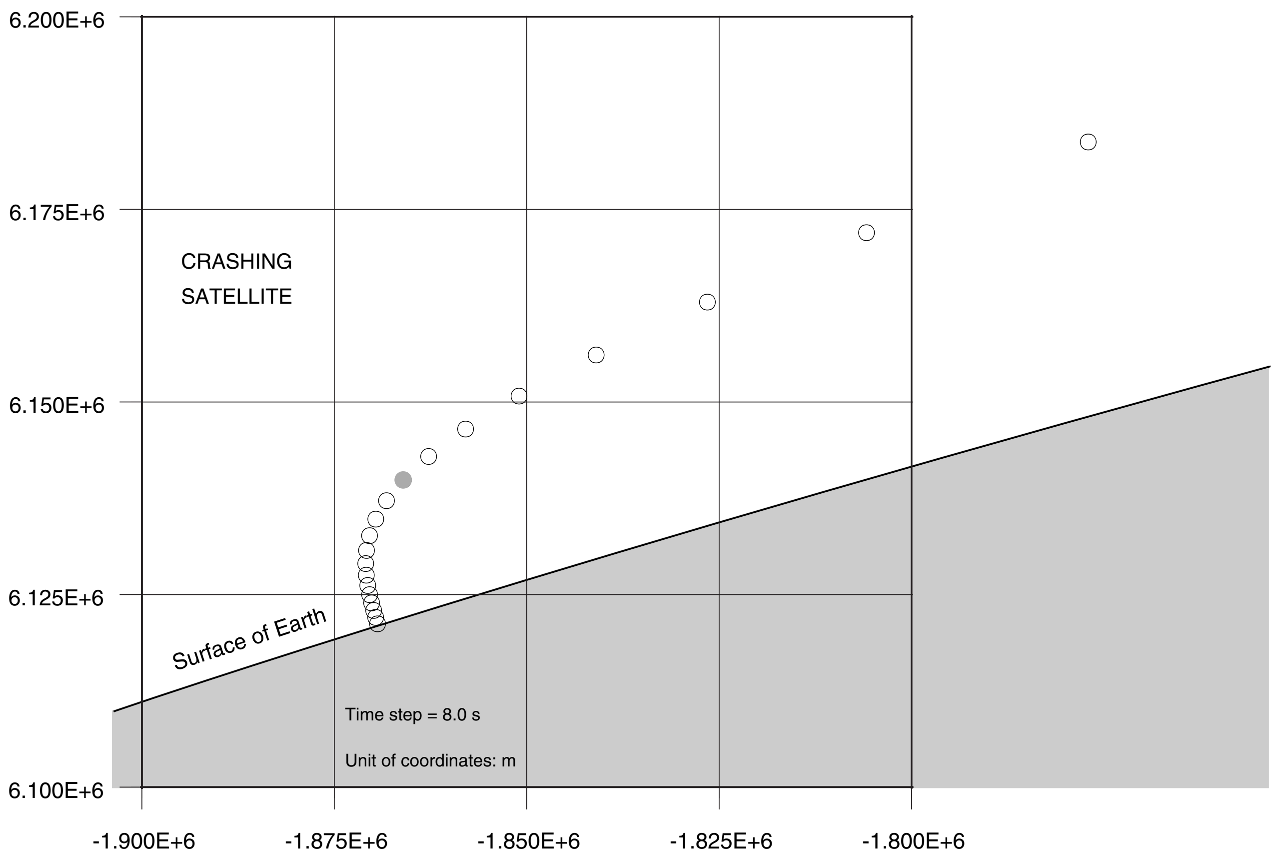
the collision. Show that it is impossible for the collision to be one-dimensional.

f. Is the collision of problem e completely elastic?

g. A light but large ball hangs from a thin string and oscillates back and forth. We have data about the motion (positions in the course of time in a stroboscopic image) and the mass of the ball. At a point where the ball is moving upward (after the lowest point), the force of the string acting upon the ball is to be determined from the data. How do you perform this task?

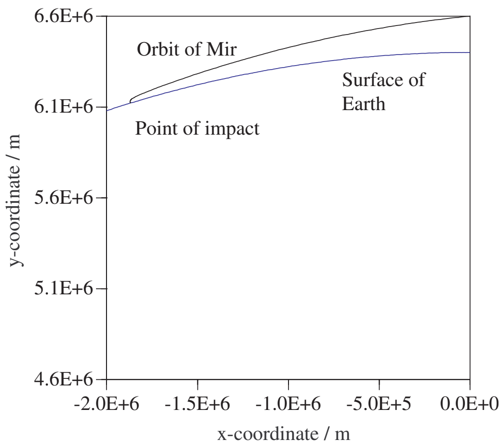
For the following problem, deliver descriptions, discussions, analyses, models, calculations… (in other words, speak intelligently in terms of physics about the situation).

4. On a horizontal air track, two gliders with repelling cow magnets attached to them move toward each other…



SOLUTIONS

1. The Mir space station's crash to earth is simulated with the help of a strongly simplified model (see figure). A greatly enlarged depiction of the crash is on an accompanying page. The mass of Mir is 60 tons. There is a strong decrease of air density when measured vertically upwards from the earth's surface.
   a. Determine the acceleration of Mir graphically at the given (gray) point shortly before impact.
   b. What is the air resistance at this point?
   c. Determine the rate at which energy is dissipated at this moment.



SOLUTION: See below.

2. Muffin cups are dropped from a certain height, and the terminal (maximal) velocity is measured (see the entries in the table). The values are the results of a series of measurements with one, two, three… cups stacked into each other.

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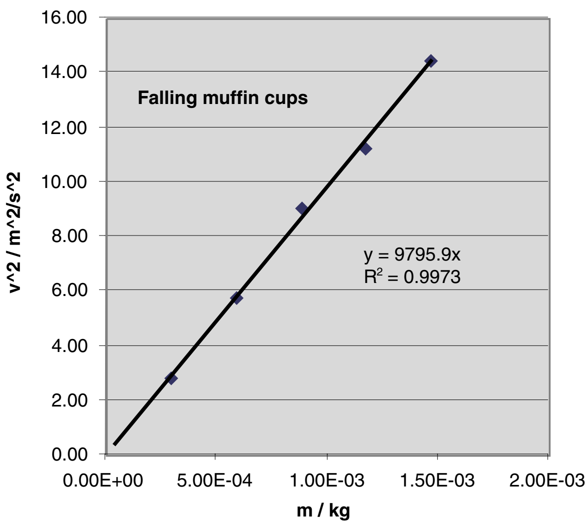
a. What kind of force law is best for this case according to the data? Is the force of air resistance proportional to the velocity, or rather to the velocity squared? Or is it proportional to the third power of the velocity? Perform a (graphical) analysis based on the data.
   Derive the theoretical relation between terminal velocity and mass of the body for the force law which you extracted from the data.

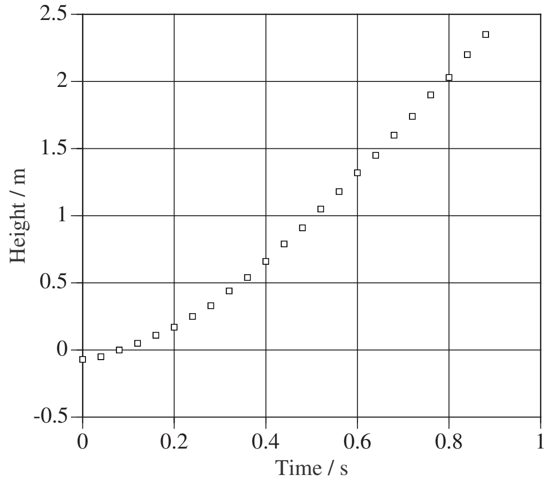
SOLUTION: Theoretical analysis. There are two forces acting on the cups: weight and air resistance. At the time when the terminal velocity has been reached, they must be equal since the accelration of the body has become equal to zero.

$$0 = F_G - F_{air}$$
$$0 = mg - kv^n$$
$$v^n = \frac{g}{k}m$$

The force of air resistance has been taken to be proportional to the n-th power of the speed. Therefore, the n-th power of the speed must be proportional to the mass of the body when the terminal speed has been reached. From our studies we expect $n = 2$. If we plot v^2 as a function of mass, we see that we get an almost straight line.



b. In the diagram below, the motion of the body consisting of 5 cups is shown. How much energy has been dissipated because of air resistance up to 0.80 s?



SOLUTION: According to the graph, the cups have fallen a distance of about 2.1 m. If there had not been any air resitsance, according to the law of balance of energy, the speed of the cups should have been

$$\frac{1}{2}mv^2 = mgh$$
$$v = \sqrt{2gh} = 6.42\frac{\text{m}}{\text{s}}$$

The actual speed at that point is 3.8 m/s according to the values in the table (or according to the slope of the function in the graph at t = 0.8 s). Therefore, the loss of energy over the distance of 2.1 m is

$$W_{loss} = \frac{1}{2}mv_{theoretical}{}^2 - \frac{1}{2}mv_{actual}{}^2$$
$$= 1.97 \cdot 10^{-2}\,\text{J}$$

3. For the following questions, give answers with explanations or detailed reasoning.
   a. A ball performs the typical motion of a projectile. Neglect air resistance. Consider a point along the trajectory where the ball is moving downward. Is there a tangential component of the acceleration of the ball?

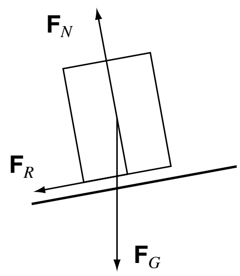
SOLUTION: There is only one force acting on the ball: gravity. Therefore, the acceleration of the ball must be a vertical vecotr. Since the path is curving downward at the point considered, there is a tangential component of acceleration in the direction of motion (the ball is getting faster).

   b. Is it possible for the acceleration of a point moving on an elliptic orbit to be zero?

SOLUTION: No, this is not possible (unless the body is at rest). As long as it moves, there always is an inward (normal) component of acceleration due to the curvature of the trajectory.

   c. A train moves along a horizontal curved track (not a circle!) while getting faster. The cars tilt inward. In the dining car, there is a glass on the table. It is slowly sliding outward (with respect to the curve of the tracks) toward the window. Draw all necessary free-body diagrams, i.e., identify all the forces acting on the glass. (Friction is possible.)

SOLUTION: There are three possible forces acting on the galss: weight, normal force of the table, and friction. Friction should be dynamic friction since the galss is sliding. The friction force is in the direction opposite to the direction of motion. A possible solution:



   d. What is the direction of the sum of all forces acting on the glass in the previous example?

SOLUTION: It's equal to the direction of the acceleration of the glass. If the glass is sliding radially outward at constant speed, the acceleration of the glass should be the same as that of the train: pointing horizontally inward and forward. Since dynamical friction cannot give us a component forward, it is to be expected that the acceleration of the glass points inward and not forward. (This means that the glass is accelerated with respect to the table.)

   e. A disc having a mass of 0.75 kg is moving on an air table toward another disc at rest (mass: 0.50 kg). There is a collision. The speed of the first disk is 0.35 m/s before the collision, and 0.25 m/s afterwards. The second disk has a speed of 0.31 m/s after the collision. Show that it is impossible for the collision to be one-dimensional.

SOLUTION: The momentum of the system is expected to be conserved. If the collision were to be one-dimensional, the conservation of momentum would hold for the scalar values:

0.75·0.35 N·s = 0.75·0.25 N·s + 0.50·0.31 N·s.

Since this is not correct, the motion cannot be one-dimensional.

f. Is the collision of problem e completely elastic?

SOLUTION: For a completely elastic collision, we expect conservation of the energy of the bodies due to their motion:

0.5·0.75·0.35^2 J = 0.5·0.75·0.25^2 J + 0.5·0.50·0.31^2 J

or 0.0459 J = 0.0475 J. This is almost an equality, certainly the results are within error margins.

g. A light but large ball hangs from a thin string and oscillates back and forth. We have data about the motion (positions in the course of time in a stroboscopic image) and the mass of the ball. At a point where the ball is moving upward (after the lowest point), the force of the string acting upon the ball is to be determined from the data. How do you perform this task?

SOLUTION: 1. From the graph we get the acceleration vector (by graphical construction). 2. We decompose the vector into tangential (a_T) and normal (a_n) components. (Alternatively, determine v, and then a_n.) 3. According to the free body diaagram, we see that there are three forces acting on the ball: weight (FG), force of string (Fs), and air resistance (Fair). Only two forces (or components) lie in the normal direction: Fs and FGn (FGn is obtained from trigonometry). 4. According to the balance of momentum for the normal direction, m·a_n = Fs – FGn. This yields Fs.

For the following problem, deliver descriptions, discussions, analyses, models, calculations… (in other words, speak intelligently in terms of physics about the situation).

4. On a horizontal air track, two gliders with repelling cow magnets attached to them move toward each other…

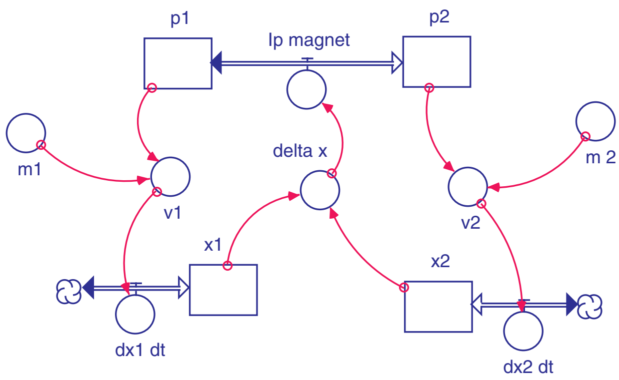
SOLUTION: If friction can be neglected, it is expected that the only force acting on a single glider is the force of the other magnet on the magnet of the glider under consideration (draw a free body diagram):

p1_dot = F_mag
p2_dot = – F_mag

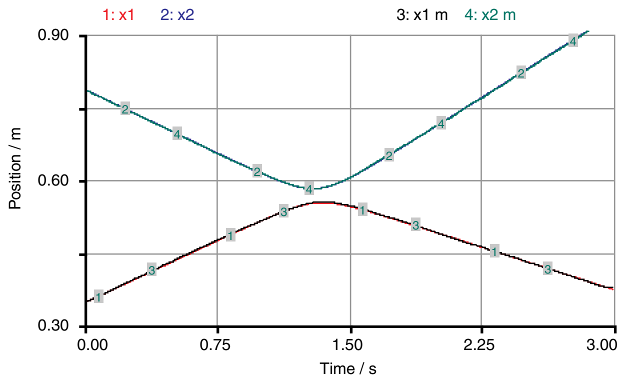
(the forces are equal but opposite due to the law of interaction; put differently, there is a momentum current from one glider to the other).

The force F_mag decreases with increasing distance of the gliders (or the magnets). For example, we might have |F_mag| = const./delta_x^5.

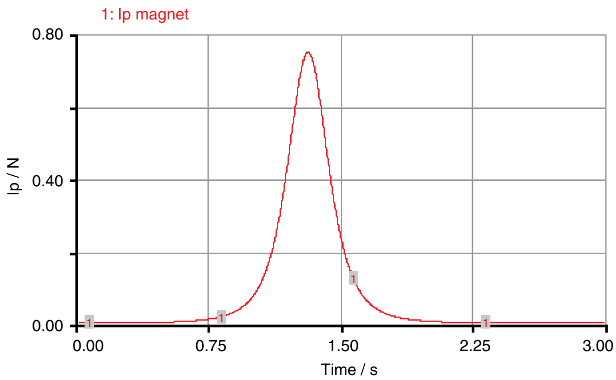
The position of the gliders is obtained by integrating the velocity which is obtained from the momentum of a body. This completes the system dynamics model:



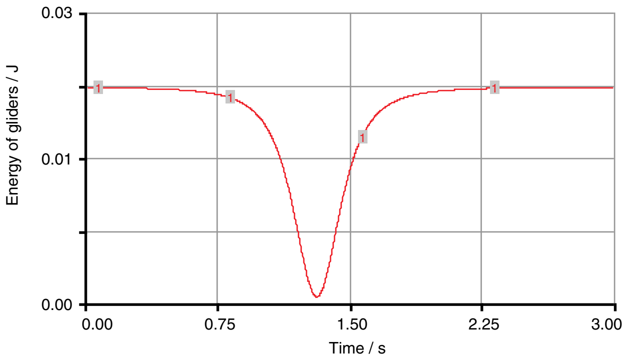
The motion of the gliders may look like this:



The force of the magnets (the momentum current from glider to glider) may look like this:



The energy of the gliders (due to their motion) may look like this:



During the collision, a part of the energy of motion (or even all of it, depending on the circumstances) is stored in the magnetic field. It is given back to the gliders after the collision (the collision with magnets is totally elastic).

SOLUTION OF PROBLEM 1.