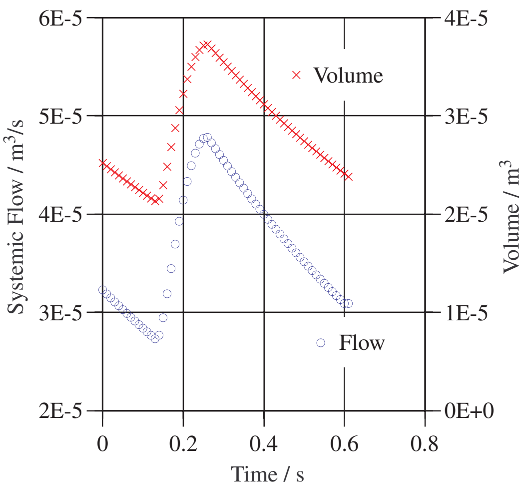
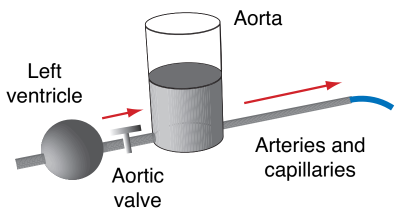
Daten des Blutvolumens und des Blutstroms aus einem Stück der Aorta (das an die linke Herzkammer angrenzt) in den Körper eines Schafs sind bekannt.



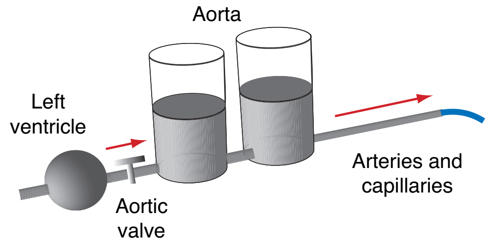
a. Benutzen Sie die Daten, um die Änderungsrate des Blutvolumens in der Aorta als Funktion der Zeit zu bestimmen (von Hand, so genau wie möglich und sinnvoll).

b. Erklären Sie mit Worten und Formeln, wie man den Blutstrom aus der linken Herzkammer in die Aorta als Funktion der Zeit bestimmt. Bestimmen Sie diesen Blutstrom dann grafisch.

c. Ein einfaches Modell für den systemischen Blutkreislauf ist das Windkesselmodell im Bild.

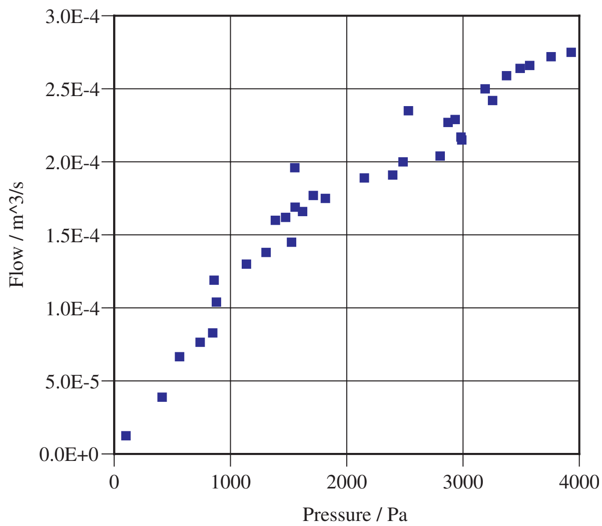


Wenn man die Aorta als zwei aneinender geschaltete Behälter modelliert, erhält man ein besseres Modell.

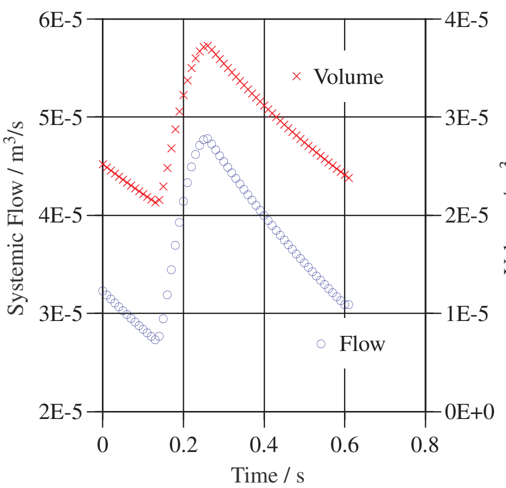


Wie gross ist die Kapazität eines einzelnen Behälters in diesem Modell im vergleich zur Kapazität des einzigen Behälters im einfacheren Modell? Gleich? Grösser oder kleiner? Wieviel grösser oder kleiner? Erklären Sie Ihre Antwort.

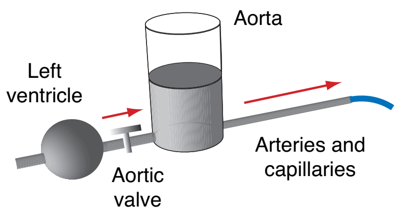
d. Aus Daten wurde der Blutstrom (aus dem Herz in die Aorta) als Funktion der Druckdifferenz zwischen Herzkammer und Aorta dargestellt. Man sieht im Diagramm zwei Bereiche, einen für laminare und einen für turbulente Strömung (Übergang von laminar zu turbulent bei 1500 Pa). Bestimmen Sie den Strömungswiderstand für den laminaren Teil und den Strömungfaktor für den turbulenten Teil. (Sie müssen einfache Kurven durch die Daten der beiden Teile legen.)



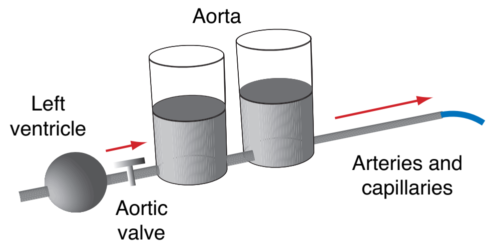
The following data is known about blood flow of a sheep: The volume of blood in a piece of the aorta (the piece borders on the left ventricle of the heart), and the blood flow out of this piece into the body.



a. Use the data to determine the rate of change of volume of blood in the piece of the aorta as a function of time. Do this by hand, as carefully as possible and reasonable.
b. Explain in words and with formulas how the flow of blood from the left ventricle into the aorta is calculated. Perform the calculation graphically.
c. The windkessel model is a simple model for the systemic circuit (see figure).

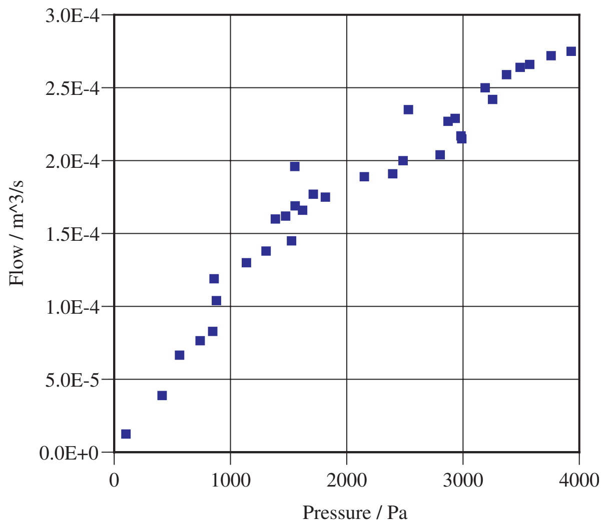


To improve the model, the aorta is represented as two connected containers (see the following picture).



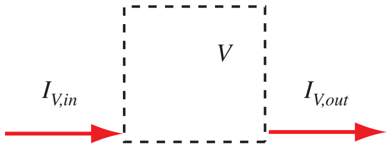
What is the capacitance of one container in this model compared to the capacitance of the single container in the simpler model? Is it equal? Is it larger? Smaller? Larger or smaller by how much? Explain your answer.

d. The data of the blood flow from the heart to the aorta is represented as a function of the pressure difference between left ventricle and aorta. In the ensuing diagram we can see two parts, one for laminar flow and one for turbulent flow (transition from laminar to turbulent flow at 1500 Pa). Determine the flow resistance for the laminar part, and the flow factor for the turbulent part. (You have to fit simple functions through the data points of the two parts.)

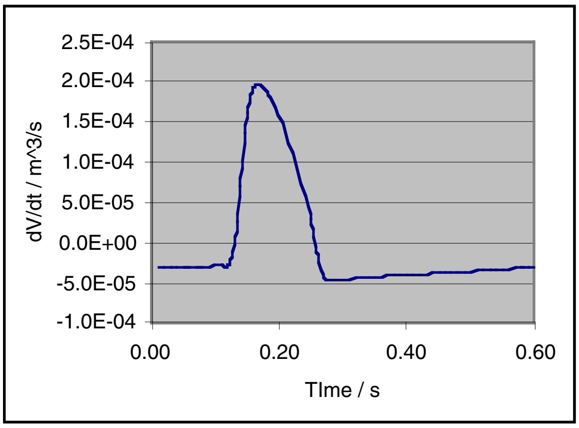


Solutions

Aorta, inflow and ourflow:

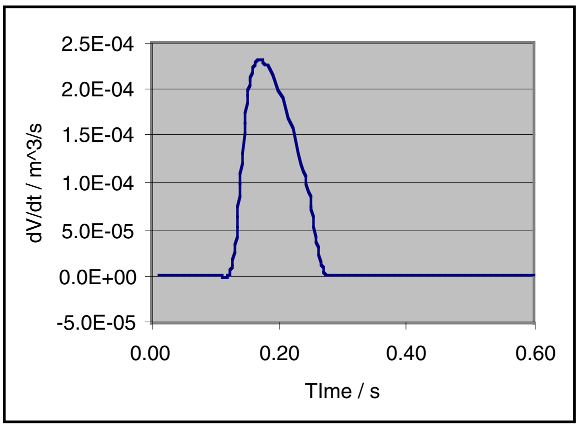


a. Determine the slopes of tangents to the V-t data and draw the result as a function of time:



b. The current flowing into this piece of the aorta is found with the help of the (instantaneous) form of the law of balance of volume:

$$\dot{V} = I_{V,in} - I_{V,out}$$
$$\Rightarrow I_{V,in} = \dot{V} + I_{V,out}$$

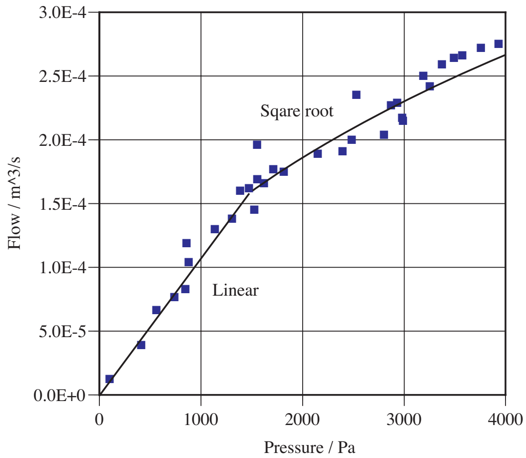


c. The real physical condition does not change: A certain quantity of blood leads to a certain rise in pressure.
This means: The pressure in each tank in the model must be the same as in the single tank model. The quantity of blood added per tank is half of what it is in the single tank model. Since

$$\Delta V = C_V \Delta p_C$$

C_V must be 1/2 that in the single tank model.

d.



Laminar flow means linear characteristic (starting at (0,0)): Slope = $1.6 \cdot 10^{-4}$ / 1500 $m^3/(s \cdot Pa)$. Therefore:

$$R_V = \frac{\Delta p_R}{I_V} = \frac{1500}{1.6 \cdot 10^{-4}} \frac{\text{Pa} \cdot \text{s}}{\text{m}^3} = 9.4 \cdot 10^6 \frac{\text{Pa} \cdot \text{s}}{\text{m}^3}$$

Turbulent: Square root function going through the point $(1500, 1.6 \cdot 10^{-4})$:

$$I_V = k\sqrt{\Delta p_R}$$
$$1.6 \cdot 10^{-4} = k\sqrt{1500}$$
$$k = 4.13 \cdot 10^{-6} \frac{\text{m}^3/\text{s}}{\text{Pa}^{0.5}}$$