Natur, Technik, Systeme

Test, April 2013

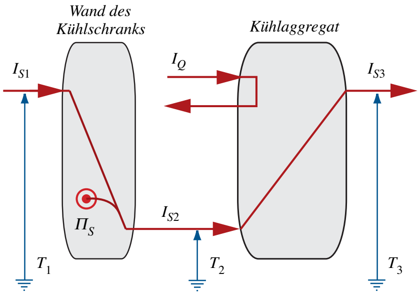
Zweites Semester WI12

Erlaubte Hilfsmittel: Bücher und persönlich verfasste Zusammenfassung. Rechen- und Schreibzeugs.

Antworten müssen begründet und nachvollziehbar sein.

Dauer des Tests: 60 Minuten.

1. Ein moderner Kühlschrank ist gut aber nicht perfekt wärmeisoliert. Er hat einen Energieleitwert (zwischen Küche und kaltem Innenraum) von 2.0 W/K. Er steht in einer Küche bei einer Temperatur von 27°C. Der Innenraum wird auf 4°C gehalten. Man im langfristigen Mittel 0.25 kWh Energie (elektrisch) für den Betrieb pro Tag.
   a. Stellen Sie sich vor, das Kühlaggregat (Wärmepumpe) des Kühlschranks könne *vollkommen ideal* betrieben werden. D.h. insbesondere, dass die im Innenraum ankommende Entropie direkt (ohne weitere Temperaturdifferenz gegenüber dem Innenraum bei 4°C) vom Kühlmittel aufgenommen und direkt an die Küche (bei 27°C, also ohne weitere Temperaturdifferenz) wieder abgegeben wird. Bestimmen Sie den Entropiestrom, der im Innenraum ankommt und die elektrische Betriebsleistung dieses Modells eines Kühlschranks. [4 P]
   b. *Modell eines realen Kühlschranks*. Die Wärmepumpe des Kühlschranks (d.h. das Kühlaggregat) arbeite reversibel, aber es braucht Wärmetauscher am kühlen und am warmen Ende. Konkret nimmt das Kühlmittel die Entropie, die aus dem Innenraum kommt, bei – 16°C auf und pumpt sie auf 47°C, von wo sie an die Küche abgegeben wird. (1) Wie gross ist der Entropiestrom, der bei der tiefen Temperatur im Kühlmittel ankommt? (2) Wie gross ist die reale elektrische Antriebsleistung in diesem Modell? (3) Wieviel ist das im Vergleich zur vom Hersteller angegebenen Antriebsleistung? [4 P]
   c. Mit welcher Rate ändert sich die Temperatur der Küche im Fall der in Aufgabe b beschriebenen Situation? Nehmen Sie an, die Küche sei perfekt thermisch isolierbar. Das Innere besteht im Wesentlichen aus 200 kg Holz (spezifische Wärmekapazität von 1700 J/(K·kg)) und 100 kg Metall (spezifische Wärmekapazität von 600 J/(K·kg)). [2 P]



Figur: Prozessdiagramm eines Kühlschranks mit ideal arbeitendem Kühlaggregat. Energiegrössen sind *nicht* eingetragen.

Natural and Technical Systems

Test, April 2013

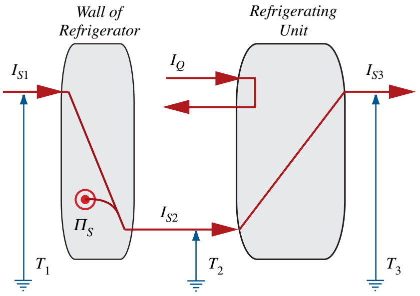
Second Semester WI12

Allowed tools: **Books and personally written summary**. Calculators and writing materials.

Answers must be explained and must be documented.

Duration of the exam: 60 minutes.

1. The thermal insulation of a modern refrigerator is strong but not perfect. Its energy conductance (between kitchen and interior cold room) is 2.0 W/K. The refrigerator is in a kitchen at 27°C and its interior is kept at 4°C. On average over longer periods, the refrigerator needs 0.25 kWh of electric energy per day (data by the manufacutrer).
    a. Imagine the refrigerating unit (heat pump) to work *completely reversibly*. This means, in particular, that the cooling fluid takes up the entropy arriving in the interior of the refrigerator (cold room) at 4°C without any additional temperature difference. The entropy is then communicated to the kitchen directly at 27°C, again without an additional temperature difference. Determine the entropy current arriving in the cold room of the refrigerator and the electric power of this model of a refrigerator. [4 P]
    b. *Model of a real refrigerator*. The heat pump (cooling unit) of the refrigerator works reversibly. However, there are heat exchangers at the cool and warm ends. The cooling fluid absorbs the entropy coming from the interior of the refrigerator at – 16°C and pumps it to 47°C from where it is emitted to the kitchen. (1) What is the entropy current arriving in the cooling fluid at the low temperature? (2) What is the real electric power in this model? (3) How much is this compared to the average electric power stated by the manufacturer? [4 P]
    c. What is the rate of change of the temperature of the kitchen according to the model in problem b? Assume the kitchen to be perfectly thermally insulated. The interior of the kitchen is made up mostly of 200 kg of wood (specific heat of 1700 J/(K·kg)) and 100 kg of metal (specific heat of 600 J/(K·kg)). [2 P]

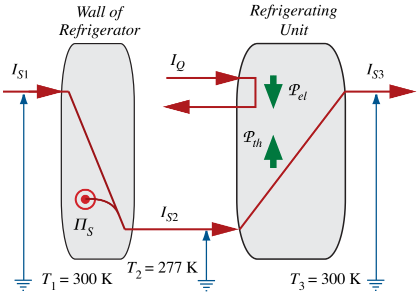


Figur: Process diagram of a refrigerator having an ideal refrigerating unit. Energy quantities have been left out of the drawing.

Solution

1. The thermal insulation of a modern refrigerator is strong but not perfect. Its energy conductance (between kitchen and interior cold room) is 2.0 W/K. The refrigerator is in a kitchen at 27°C and its interior is kept at 4°C. On average over longer periods, the refrigerator needs 0.25 kWh of electric energy per day (data by the manufacutrer).

   a. Calculate the energy current through the wall of the refrigerator. This current is constant across the wall. From this we calculate the entropy current at T2:

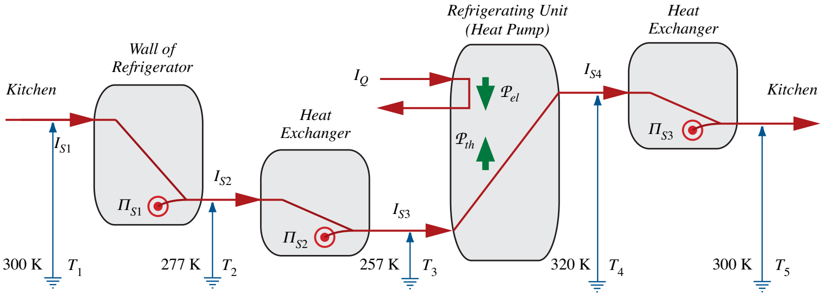


$$
\begin{aligned}
I_{E1} &= G_E\left(T_1 - T_2\right)\\
I_{S2} &= \frac{1}{T_2} I_{E1} = \frac{1}{T_2} G_E\left(T_1 - T_2\right)\\
&= \frac{2.0\cdot\left(300-277\right)}{277}\frac{\text{W}}{\text{K}} = 0.166\frac{\text{W}}{\text{K}}
\end{aligned}
$$

   This entropy current must be pumped to 300 K. The power to do this is

$$
\begin{aligned}
\mathcal{P}_{el} &= \mathcal{P}_{th}\\
\mathcal{P}_{th} &= \left(T_3 - T_2\right) I_{S2}\\
&= \left(300-277\right)\cdot 0.166\,\text{W} = 3.82\,\text{W}
\end{aligned}
$$

   b. *Model of a real refrigerator*. The heat pump (cooling unit) of the refrigerator works reversibly. However, there are heat exchangers at the cool and warm ends. The cooling fluid absorbs the entropy coming from the interior of the refrigerator at – 16°C and pumps it to 47°C from where it is emitted to the kitchen.



   The energy current from the kitchen going trhough the wall of the refrigerator now ends up in the cooling fluid at 257 K. Therefore:

$$
\begin{aligned}
I_{E1} &= G_E\left(T_1 - T_2\right)\\
I_{S3} &= \frac{1}{T_3} I_{E1} = \frac{2.0\cdot\left(300-277\right)}{257}\frac{\text{W}}{\text{K}} = 0.179\frac{\text{W}}{\text{K}}
\end{aligned}
$$

The power of pumping this current to 320 K is

$$\begin{aligned}\mathcal{P}_{el} &= \mathcal{P}_{th} \\ \mathcal{P}_{th} &= \left(T_4 - T_3\right) I_{S3} \\ &= \left(320 - 257\right) \cdot 0.179\,\mathrm{W} = 11.3\,\mathrm{W}\end{aligned}$$

The average electric power specified by the manufacturer is

$$\mathcal{P}_{el} = 0.25 \frac{kWh}{d} = \frac{0.25 \cdot 3.6 \cdot 10^6}{24 \cdot 3600}\,\mathrm{W} = 10.4\,\mathrm{W}$$

This is slightly less than the power calculated from the endoreversible model.

c. What is the rate of change of the temperature of the kitchen according to the model in problem b?

The energy dissipated in the kitchen equals the electric energy delivered to the room. Therefore:

$$\begin{aligned}\mathcal{P}_{diss} &= \mathcal{P}_{el} \\ \dot{E}_{kitchen} &= \mathcal{P}_{diss} \\ \dot{E}_{Kitchen} &= C_{Kitchen} \dot{T}_K \\ C_{Kitchen} &= m_1 c_1 + m_2 c_2 \\ &= 200 \cdot 1700 \frac{\mathrm{J}}{\mathrm{K}} + 100 \cdot 600 \frac{\mathrm{J}}{\mathrm{K}} = 4.0 \cdot 10^5 \frac{\mathrm{J}}{\mathrm{K}} \\ \dot{T}_K &= \frac{\dot{E}_{Kitchen}}{C_{Kitchen}} = \frac{\mathcal{P}_{el}}{C_{Kitchen}} = \frac{11.3}{4.0 \cdot 10^5} \frac{\mathrm{K}}{\mathrm{s}} = 2.8 \cdot 10^{-5} \frac{\mathrm{K}}{\mathrm{s}}\end{aligned}$$