NTSY2, Test 1

April 2017, WI16a-b

Time for test: 60 Minuten.
Answers must be explained and must be documented.
Allowed tools: Personally written summary of up to 6 pages (plus 6 pages from NTSY1. Book: The Dynamics of Heat. Calculators and writing materials.

Model

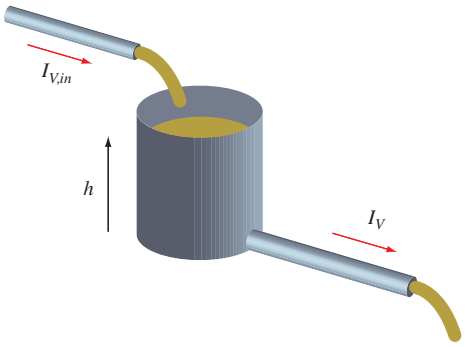

Figure 1: Model of the lake with inflow and discharge of water.

There was an accident at a chemical factory that released a toxic substance. The substance is carried by water into a nearby lake. That would not be much of a problem since the substance decays into nontoxic substances over the course of time. However, a farmer has been taking water from the discharge of the lake illegally in order to irrigate his fields. At time $t = 4 \cdot 10^4$ s the accident is discovered and both the inflow and discharge of water from the lake are stopped.

We are modeling the lake as a straight-walled tank, see Figure 1. The water in the tank (lake) is always perfectly mixed so that the concentration of the toxic substance is the same (equal to c) at every point in the lake.

The volume current of water *into* the tank is constant up to the moment $t = 4 \cdot 10^4$ s. Starting at that point in time, the inflow is equal to zero.

The volume current *out of* the tank will be modeled as being laminar having a constant hydraulic conductance. Starting at $t = 4 \cdot 10^4$ s, the volume current *out of* the tank will be equal to zero.

The flows of substance into and out of the tank will be modeled as being convective. The concentration of the toxic substance flowing with water *into* the lake is constant.

The level of water and the concentration of the toxic substance in the tank are shown as functions of time in Figure 2.

Data

Hydraulic conductance: $2 \cdot 10^{-4}\,\mathrm{m}^3/\mathrm{Pa/s}$
Cross section of the tank: $10^4\,\mathrm{m}^2$
Density of the liquid: $1000\,\mathrm{kg/m}^3$
Gravitational field: $10\,\mathrm{N/kg}$

Questions/Problems

1. Formulate the law of balance of volume. [0.5 P]
2. Formulate the law of balance of amount of substance of the toxic substance. [0.5 P]
3. Explain the form of the curve for $h(t)$. Why does the level go down at the beginning even though water is flowing into the tank? What is the form of this part of the curve? Why is h constant after this first phase? [1 P]
4. Explain the form for $c(t)$. Why does the concentration rise fast at the beginning? Why does the curve level off after this? Why does it go down again later? What is the form of the last section of the function? [1 P]
5. Determine the rate of change of concentration right after inflow and outflow have been stopped. [1 P]
6. Determine the decay constant of the reaction of the toxic substance. [1 P]
7. Determine the rate of change of concentration at $t = 0$. [1 P]
8. Determine the concentration of the substance in the water flowing into the tank. [4 P]

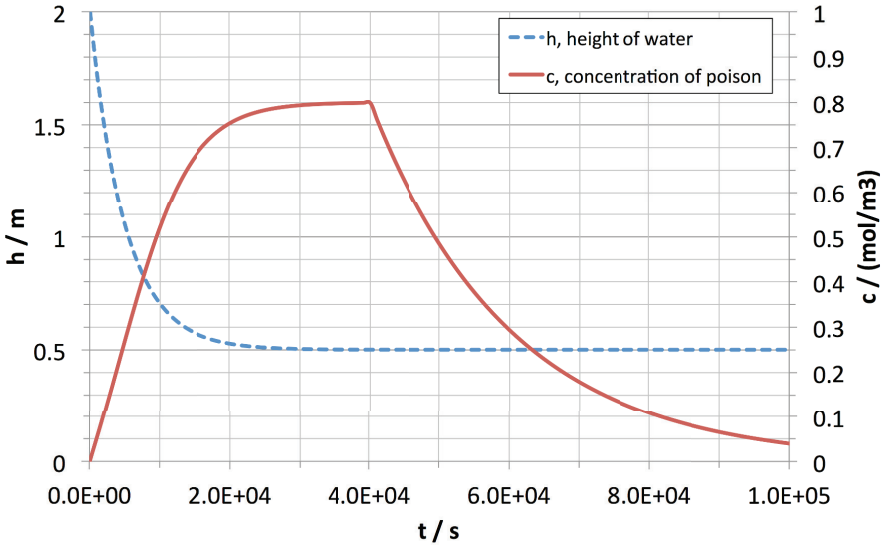

Figure 2: Level of water h and concentration c in the tank as functions of time.