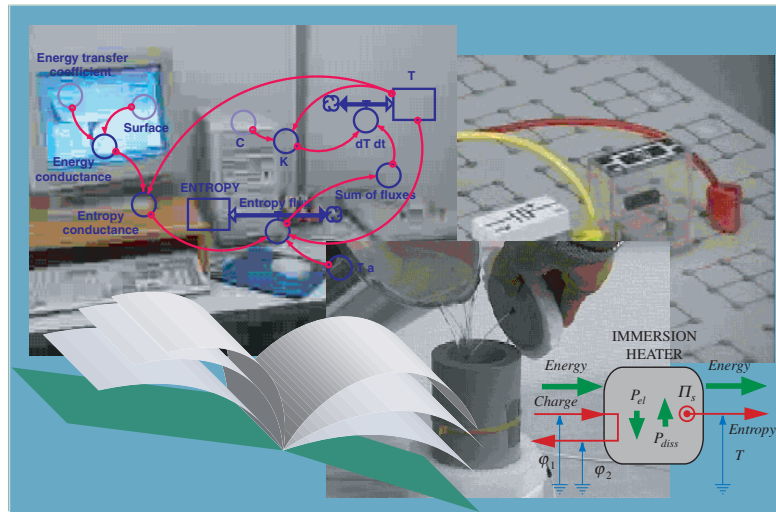# PHYSICS

AN INTEGRATED SYSTEMDYNAMICS LEARNING ENVIRONMENT

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THE DYNAMICS OF HEAT

GOALS

The Dynamics of Heat opens the world of thermal dynamical processes. Part IV provides you with activities and study materials, experiments and models so you can actively follow a given—or your own—path to explore the world of thermodynamics.

Thermodynamics is the science of heat and temperature, and their relation to energy. The everyday concept of heat turns out to be the modern quantity called entropy, whereas temperature measures hotness, i.e., how warm bodies are.

In this part you will get to know the fundamental concepts and tools, and you will be able to learn about heating and cooling of simple materials such as water and stones, and about melting and evaporation.

In the end you should be able to describe and model thermal dynamical processes, and to compare your models to reality.

LIST OF ASSIGNMENTS AND EXPLORATIONS

SECTIONS	ASSIGNMENT 1	ASSIGNMENT 2	EXPLORATION
I.2 Thermal Phenomena	Describe heating of ice – water – steam, compression of air, and production of entropy in a fire	Determine entropy production rate and entropy produced in electric heating of water	
I.2 Hotness and Temperature	Gauging resistive thermometer	Gas thermometer: pressure temperature relation for air	Thermoelectric elements
I.3 Entropy and Temperature in Simple Materials	Electric heating of water in perfectly insulated container	Electric heating of glycol in perfectly insulated container	
I.4 Entropy and Energy in Heat Transfer	Cooling of water in a thin-walled container	Heating of water in insulated container in warm environment	Time constants of a thermometer
I.5 Heat Transfer and Entropy Production	Two bodies of water in thermal contact	Cold copper cylinder in hot water	
I.6 Melting and Evaporation	Evaporating water in an open insulated container	Warming and melting of ice in a test tube placed in hot water	
P.1 Cooling of Water	Cooling of water in a cold, thick-walled PVC container		
P.2 Latent Entropy Storage	Wax in tubes placed in water which is heated slowly		

THERMAL PHENOMENA

Thermodynamics is the science of heat and hotness, and of how bodies and other physical systems respond to heat.

Heat is the quantity responsible for making bodies warm, or for melting ice and evaporating water, or for making air expand. You may imagine it to be like an invisible "fluid" (the "heat substance") which resides in bodies and which can flow into and out of systems (Figure 1). It behaves like charge in electricity or amount of substance in hydraulics; it is the extensive thermal quantity. In contrast to electric charge heat is not conserved: it can be created but not destroyed. This important property reflects the irreversibility of processes.

Hotness describes how warm bodies are. Hot and cold—which are different degrees of hotness—are measured by temperature. Hotness (or temperature) are the intensive thermal quantity, and temperature differences take the role of the thermal driving force.

Heat can do work (Figure 2). In other words, it releases energy if it flows from a hotter to a colder place (like water flowing from a higher to a lower place; Figure 3). The observation that heat can drive other processes establishes the relation between heat, temperature and energy.
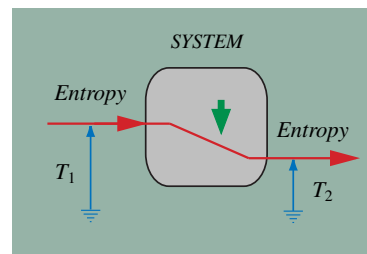

Fig.1

Fig.2

Fig.3

The technical term for what we have called "heat" is entropy. Entropy is the substancelike thermal quantity.

EXPERIMENTS

In this section we will study some experiments and situations which demonstrate the properties of the fundamental thermal quantities—namely—heat (entropy) and hotness (temperature).

From the multitude of thermal process we study the melting of ice (with subsequent warming and evaporation of water), the compression of air, and the burning of a fuel. Then we study the balance of entropy for a building, and finally we will look at entropy production in an electric immersion heater. These processes teach us about entropy, temperature, and the role of energy in thermal processes.

Open WorkBook for this Section

Assignment and Experiment 1

Melting of ice, warming and evaporation of water; compression of air; burning of a fuel

Assignment and Experiment 2

Entropy production in an electric immersion heater heating water

DESCRIBING AND EXPLAINING SOME THERMAL PHENOMENA

Four different processes are studied here: phase changes due to heating of ice and water, boiling and vaporizing of water, the compression of air, and the burning of a fire inside a room. These phenomena teach us about some important properties of entropy and temperature.

General Assignment

Some experiments demonstrate the phenomena discussed in this assignment.

a. **Study the experiments or situations.**
b. **Explain what you see in terms of entropy and temperature; use the law of balance of entropy where appropriate.**

Specific Tasks

a. **Sketch the temperature–entropy diagram of the substance going from ice to water to steam. Represent the laws of balance of entropy for water and steam (separately) during boiling in the form of a system dynamics diagram.**
b. **Sketch the TS diagram for the compression of air without heating or cooling; formulate the law of balance of entropy.**
c. **Sketch the TS diagram for a sealed room with a fire burning inside; formulate the law of balance of entropy.**

QUESTIONS & PROBLEMS

Q1. Why do melting (of ice) or vaporization (of water) demonstrate the difference between heat (entropy) and temperature so clearly? Why is this difference harder to see with the heating of water (or a stone)?

Q2. What curve represents the condensation of water vapor in the TS diagram?

Q3. A hot piece of iron is cooling in a cold environment. What are the TS diagrams for the piece of iron and the environment during this process?

Q4. Where does entropy come from and go to in the (steady-state) operation of an electric heater?

Q5. Water is heated electrically in a thin-walled container. It is open at the top so vapor can escape. What is the form of the law of balance of entropy of water and vapor taken as a single system? How is the law different if we only look at the water?

Q6. Which properties of entropy are reflected in its law of balance?

Q7. For the processes undergone by a system, the change of entropy and the net amount of entropy produced in the system are known. How can you calculate the net amount of entropy exchanged between the system and the environment?

Q8. What is the meaning of the terms "heating" and "cooling"? Is the meaning the same as for "getting warmer" and "getting colder"?

Q9. What is the TS diagram for the expansion of air in a cylinder with piston? Assume there to be no heating and cooling. Is the diagram different for reversible and irreversible expansion?

Q10. Is it possible for air to get hotter when its entropy decreases?

Q11. Consider two ways of making a room warmer. In the first, entropy is produced by a fire inside the room. In the second, entropy flows in from a hot environment. Are the TS diagrams for the room different for these situations? Are the forms of the law of balance of entropy different?

Q12. What principles did you learn by doing Assignment 1?

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P1. A glass sphere exposed to sunlight is getting warmer. Its entropy increases at the rate of 0.10 W/K. At the same time it looses entropy to the colder environment a a rate of 0.060 W/K. What is the rate of production of entropy due to absorption of light?

P2. The rate of change of entropy of a body changes uniformly from 0.10 W/K to 0.050 W/K in 20 s. What is the change of entropy in this period?

PRODUCING ENTROPY IN THE ELECTRIC HEATING OF WATER

During the heating of water with the help of an immersion heater, the water temperature, the voltage and the electric current are monitored. The entropy production rate and the total entropy produced are to be determined.

General Assignment

Experiment RAVEL_IV.1-2 is to be investigated, and the entropy production in the heater is to be determined.

  a. **Start by investigating the real experiment (RAVEL_IV.1-2).**
  b. **Perform an analysis of the operation of the electric heater.**
  c. **Find the entropy production rate for the heater.**

Specific Tasks

  a. **Represent the entropy production rate in the electric heater in terms of the measured data in a system dynamics model.**
  b. **Introduce the measured data to be used in your model.**
  c. **Add the calculation of the total entropy produced to your model.**

QUESTIONS & PROBLEMS

Q1. What is the steady state process diagram of an immersion heater?

Q2. What is the meaning of dissipation rate? How is it related to the rate of production of entropy?

Q3. What is the dissipation rate of an electric heater?

Q4. Assume that the entropy production rate in a process is known as a function of time. How can the total entropy produced be determined from this information?

Q5. In the experiment, water is heated by an electric heater. Does this mean that the change of the entropy of the body of water is equal to the total entropy produced by the heater?

Q6. How do you determine the entropy production rate due to the rubbing of your hands?

Q7. Sunlight is absorbed by the absorber of a solar hot water collector. How do you determine the entropy production rate as a consequence of the process of absorption of light?

Q8. Why does the fact that entropy can be produced require that the hotness has a point of absolute zero?

Q9. What principles did you learn by doing Assignment 2?

———————————————

P1. An immersion heater having an electric power of 200 W is heating water at 30°C. In the next 100 s, the temperature of the water rises uniformly to 50°C. How much entropy has been produced by the heater?

P2. In experiment RAVEL_IV.1-2, determine the entropy produced in the first half of the period of heating, then repeat the determination for the second half. Is the entropy produced in these two half periods equal? If it is not, why not?

P3. A hydroelectric power plant (artificial lake, turbines, generator) has an overall efficiency of 82%. The output electric power is 200 MW. Estimate the entropy production rate in this system.
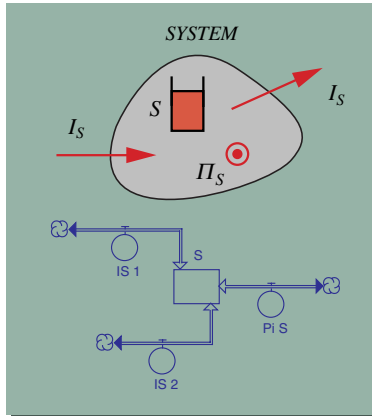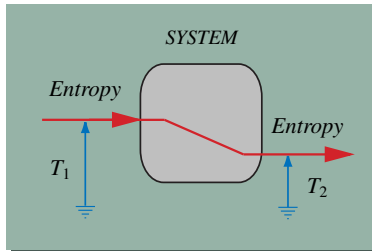
Fig.4

Fig.5

Thermal phenomena can be understood in terms of entropy and temperature. Entropy is the technical term for what in everyday life we would call heat. It is the substancelike thermal quantity that obeys a law of balance:

$$\dot{S} = I_{S,net} + \Pi_S \qquad (1)$$

When thermal physics was founded about 200 years ago, scientists thought of heat as a kind of invisible "substance" which they called caloric. Our modern concept of entropy is close to caloric, with the important difference that we know that entropy can be produced but not destroyed. This property of entropy reflects the irreversibility of natural processes. The properties of entropy are:

a. it can be stored in bodies
b. it can flow into and out of bodies
c. it can be produced in irreversible processes (rubbing, burning, electric conduction, absorption of light…)
d. it makes bodies warm, or is responsible for melting and evaporation, or it lets air expand…
e. it can work, i.e., it can release energy and drive other processes (heat engines),
f. or it can be pumped if energy is available (heat pump).

Temperature is the measure of hotness, i.e., it tells us how warm a body is. Hotness is a one-dimensional ordered manifold (like a string of beads) leading from cold to hot. Hotness has a lowest possible value (nothing can be colder than "absolutely cold"). Therefore it makes sense to introduce a temperature scale which starts with a value of zero at this lowest point (absolute temperature scale or Kelvin scale; see Section IV.2).

Temperature is the intensive thermal quantity. Temperature differences serve as the "driving force" of the flow of entropy (by itself, entropy flows from hotter to colder places; Fig.5). In other words, temperature is the thermal potential.

The observation that heat can do work establishes the relation between entropy, temperature, and energy. In simple terms, entropy is the energy carrier in thermal processes (in heating and cooling), and temperature is the thermal level (load factor).

Entropy releases energy when it flows from a higher to a lower thermal level (Fig.6). Sadi Carnot compared heat (he called it caloric) to water which, when falling down, can drive an engine. Taking this idea we postulate that the thermal power (the rate at which energy is released when entropy falls from a higher to a lower temperature) is given by

$$P_{th} = -(T_2 - T_1)I_S \qquad (2)$$

Entropy can be pumped like water, and as in the case of water the rate at which energy is needed to pump a current of entropy is also given by Eq.2.

A special case of pumping of entropy occurs when entropy is produced in an irreversible process (Fig.7). If it is produced, the entropy appearing in a system has to be lifted from absolute zero to the temperature of the body. Therefore the relation between the rate at which energy is used to produce entropy (which is called dissipation rate), the entropy production rate and the temperature of the system is

$$D = T\Pi_S \qquad (3)$$

As in all other physical processes, the energy released does not appear out of the blue. It is usually brought to the system by a flow of entropy (Fig.8). In conductive entropy transport we have
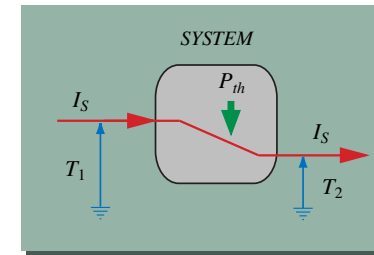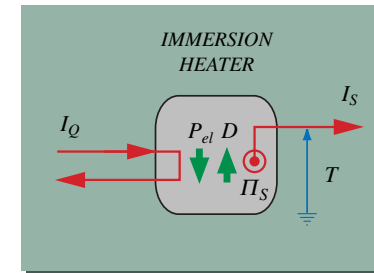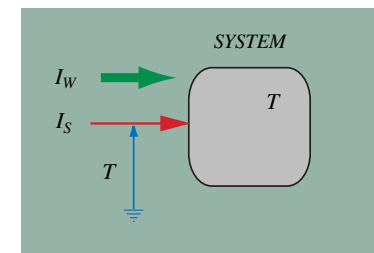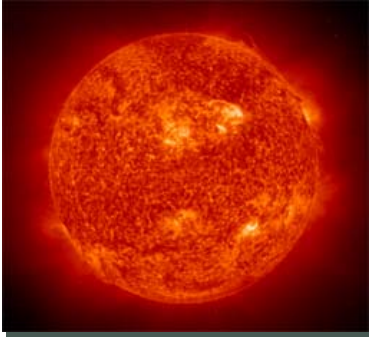
$$I_W = TI_S \qquad (4)$$

Fig.6

Fig.7

Fig.8

HOTNESS AND TEMPERATURE

Hotness is the measure of how warm a body is. You may imagine hotness to be like a string with beads where the beads represent values of hotness from very cold to cold, luke-warm, warm, hot, very hot which are ordered along the string. Or you may think of it as a road leading from cold to hot. Mathematicians call this an ordered one-dimensional manifold.

Temperature is like a coordinate on the road of hotness. It introduces values with which you can find the same hotness again, just as you can find a point along a street if the coordinate is given. As with coordinates, you can introduce many different scales of temperature on the "string" of hotness.

Temperature can be measured with the help of any substance or physical system whose properties change if their temperature changes. Such thermometric properties are the length of rods, the volume of liquids, the pressure of gases, the electric resistance of conductors, the voltage of thermocouples, thermal radiation, and many more.

An important point concerns the question if there is a temperature which is independent of the thermometric substance. The answer is yes: the pressure of an ideal gas can serve as such a measure, and the temperature based on this thermometer (Figure 2) is used as the thermal potential (see Section 1, and Figure 3).
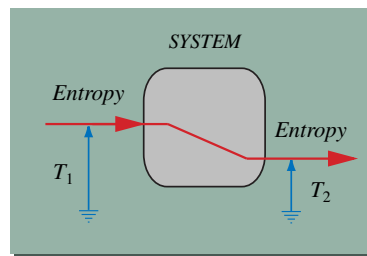

Fig.1

Fig.2

Fig.3

EXPERIMENTS

In this section we will investigate the use of an electric resistor and air as thermometric substances. We start with mercury thermometers and the Celsius temperature scale based upon them.

By putting an electric resistive element together with a mercury thermometer into a liquid at different temperatures allows us to calibrate the resistor and use it as a new thermometer.

Measurements of the pressure of an amount of air at constant volume as a function of (Celsius) temperature reveals an absolute temperature upon which the Kelvin scale is based.

Open WorkBook for this Section

Assignment and Experiment 1

Calibrating an electric resistive element (a piece of wire made from iron) using a mercury thermometer

Assignment and Experiment 2

The gas thermometer (measuring the pressure of air at constant volume as a function of Celsius temperature)

Part IV: THE DYNAMICS OF HEAT

CALIBRATING A RESISTIVE THERMOMETER

An electric resistive element is placed in water together with a mercury thermometer. The water is heated to different temperatures. As the temperature changes, the voltage and the electric current for the resistor are measured. In this manner the resistance-temperature characteristic curve of the conductor is determined.

General Assignment

Experiment (RAVEL_IV.2-1) is used to find the resistance-temperature relation for an electric conductor. In this manner, the material is calibrated and can be used as a thermometer.

    a. **Start by investigating the real experiment (RAVEL_IV.2-1).**
    b. **Perform an analysis of the experimental setup.**
    c. **Find the resistivity-temperature relation for the electric conductor.**

Specific Tasks

    a. **Find the resistance of the electric conductor for each data point.**
    b. **Determine the best (linear or nonlinear) fit of the resistance values as a function of Celsius temperature.**
    c. **Determine the standard form of this interpolation function.**

QUESTIONS & PROBLEMS

Q1. Does a thermometer measure the temperature of the body it is brought in contact with, or does it measure its own temperature?

Q2. How does a thermometer work?

Q3. Could water be used in a thermometer?

Q4. How is the resistance of the electric thermometer determined? How do you determine the resistivity of the material?

Q5. How can you be sure that the temperature of the resistive element is the same as the one read from the mercury thermometer?

Q6. How does a mercury thermometer or a resistance thermometer respond to a sudden change of temperature?

Q7. What is the meaning of the (linear) temperature coefficient of resistivity?

Q8. Is the quadratic temperature coefficient of resistivity equal to the square of the linear coefficient?

Q9. The resistance-temperature relation for the material used in the experiment is non-linear. Could this mean that the mercury thermometer does not really measure a temperature which is independent of the thermometric substance? In other words, could it mean that the mercury thermometer is nonlinear on a "true" temperature scale?

Q10. What principles did you learn by doing Assignment 1?

P1. For the experimental data in the range of 27°C to 56°C, a quadratic approximation of the resistance is R = 143.44 − 6.4063*T + 0.117*T^2, where R is in Ohm and T is measured in degrees Celsius. What are the linear and the quadratic temperature coefficients if the reference temperature is taken to be 30°C?

P2. What is the resistance of 100 m of copper wire having a radius of 0.10 mm at a temperature of 200°C?

P3. The time constant of a resistance thermometer placed from air in water is 5 s. It is taken from 20°C air and placed in 50°C water. How long does it take for the temperature of the thermometer to be within 1°C of the final temperature?

Part IV: THE DYNAMICS OF HEAT

THE GAS THERMOMETER AND THE IDEAL GAS TEMPERATURE

Air is heated in a glass bulb, and its pressure is measured as a function of temperature. Data of Experiment RAVEL_IV.2-2 are to be used to determine the relation between the pressure and the temperature and to find the temperature at which the pressure vanishes.

General Assignment

Experiment (RAVEL_IV.2-2) is used to find the pressure-temperature relation for air at constant volume. With the help of this relation, the ideal gas temperature is introduced upon which the (absolute) Kelvin scale is based.

   a.   **Start by investigating the real experiment (RAVEL_IV.2-2).**
   b.   **Perform an analysis of the gas thermometer.**
   c.   **Find the pressure-temperature relation for air.**

Specific Tasks

   a.   **Find the pressure of the air from the measured data. Determine the best (linear) fit of the pressure values as a function of Celsius temperature.**
   b.   **Determine the (Celsius) temperature at which the (linear) relation between pressure and temperature intersects the temperature axis.**
   c.   **Base a new temperature scale upon the gas thermometer (Kelvin scale). The scale is to have its zero point where the pressure of the gas would vanish, and a unit in this scale is equal to the unit of temperature in the Celsius scale.**

QUESTIONS & PROBLEMS

Q1. In the gas thermometer, which property of the air is used to determine the (ideal gas) temperature?

Q2. How is the pressure of the air in Experiment RAVEL IV.2-2 determined? Does the ambient pressure play a role?

Q3. Can you be certain on the basis of the experiment that the temperature at which the pressure of the air vanishes is the lowest possible temperature in nature?

Q4. Will the pressure of air really go to zero at the temperature derived from the experiment?

Q5. If there was twice as much air in the bulb than in the actual experiment, what would the pressure-Celsius temperature relation look like?

Q6. How much helium would have to be put into the bulb so that the pressure-temperature relation was is exactly the same as the one found in the experiment?

Q7. What principles did you learn by doing Assignment 2?

_____

P1. If we had made an error in the reading of the ambient pressure, would we still get a linear relation between pressure of the air in the bulb and the (Celsius) temperature? Would we still get the same value for the zero of the temperature scale?

P2. If we had 0.010 mole of nitrogen gas in the gas thermometer, what would its pressure be at a temperature of 0°C? The volume of the bulb in the gas thermometer is equal to 0.10 liters.
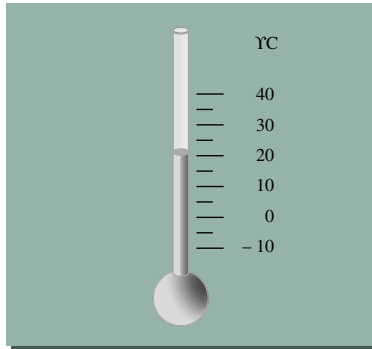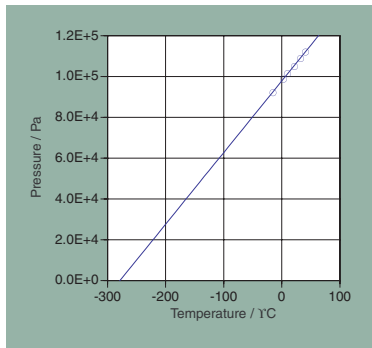
Temperature is the coordinate on the hotness manifold. It can be determined practically by any substance whose property or properties change if its hotness changes. A property which changes with temperature is called a thermoscopic property. Using such a thermoscopic property, a thermometer and an associated temperature scale can be introduced. Such a scale depends upon the material used in the thermometer. A first problem of thermometry consists of finding a useful material to introduce a first empirical temperature scale.

The second challenge of thermometry is to find the value of the lowest possible temperature in the temperature scale introduced in step one.

On the other hand, thermodynamics rest upon a temperature scale which is independent of any material and can be used as an absolute scale. Such an absolute scale is needed to express the relations between entropy and energy (Section IV.1). This constitutes the third challenge of thermometry.

A first practical empirical temperature can be based upon the mercury thermometer (Fig.4): the change of volume of the liquid is used as the thermometric property. Using such a thermometer, we can introduce the Celsius scale: the temperautre of the freezing point of water is set to 0°C, and 100°C is the temperature of vaporization of water at an air pressure of 1 bar. The Celsius temperature are defined to be a linear function of the volume of the thermometric liquid.

It became known early in thermodynamics, that an air thermometer (Fig.2) would yield a very useful temperature scale. For a dilute gas, the gas pressure is pretty much a linear function of the temperature of a mercury thermometer (Fig.5):

$$P = P_o(1 + \beta\theta) \quad \text{where} \quad \beta = 1/\theta_o \qquad \textbf{(1)}$$

beta is called the linear temperature coefficient of pressure.

Fig.4

Fig.5

It is found that for all types of dilute gases the pressure vanishes at the same value of theta_o = – 273°C. That means that all dilute gases have the same temperature coefficient of pressure, namely, beta = 1/273°C. Therefore we can introduce a new temperature T*

$$T* = \theta + \theta_o \quad \text{where} \quad \theta_o = 273°C \qquad \text{(2)}$$

which allows us to express the gas pressure in a simpler form:

$$P = \frac{P_o}{\theta_o} T* \quad \text{where} \quad \frac{P_o}{\theta_o} = \frac{nR}{V} \qquad \text{(3)}$$

T* is called the (empirical) Kelvin temperature. A gas that satisfies the relation PV = nRT* is called an ideal gas, and T* is also called the ideal gas temperature.

An important result of the thermodynamics of gases is that the ideal gas temperature can be used as an absolute temperature, i.e., one that is independent of the thermoscopic material. Therefore, T* = T, which is our measure of the thermal potential.

Many other materials and material properties are used for thermometry. For us, the electric resistance of electric conductors serves as a practical means for determining temperatures. Such thermometers can be calibrated using either mercury thermometers or the air (gas) thermometer (Fig.6). Concrete measurements can be approximated by different functions, such as a quadratic one:

Fig.6

$$\rho_{el} = \rho_{el,ref}\left(1 + \alpha\left(T - T_{ref}\right) + \beta\left(T - T_{ref}\right)^2\right) \qquad \text{(4)}$$

alpha and beta are called the linear and quadratic temperature coefficients, respectively. T_ref is an arbitrary reference temperature.
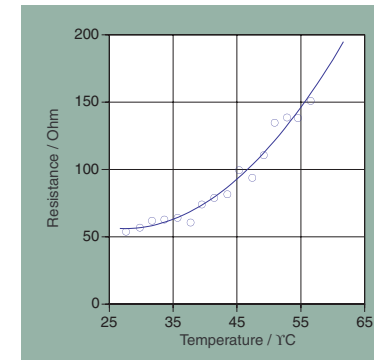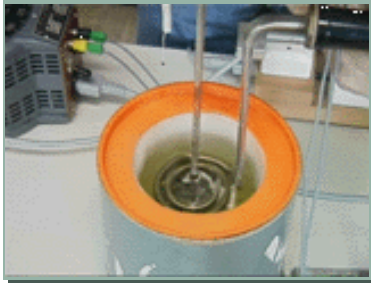
Fig.1

Entropy can have different effects upon materials, such as expanding them, letting them melt or evaporate (vaporize), or making them warmer. Here we will study cases where the only response to an increase of the entropy of the body is an increase of its temperature. For this we will heat some liquids and measure their temperature (Fig.1).

Different materials are distinguished by how strongly they respond to an increase of their entropy (Fig.2). For a material there is a unique relation which we can represent in the form of a temperature–specific entropy diagram (T-s diagram, see Fig.3). The specific entropy is the entropy per mass.

If we know the T-s relation, we can calculate the temperature from specific entropy. Or we can determine how much entropy we need to effect a certain change of temperature of the body.
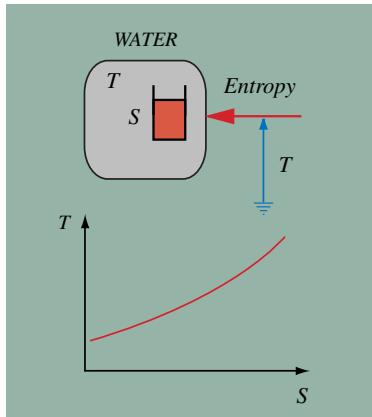
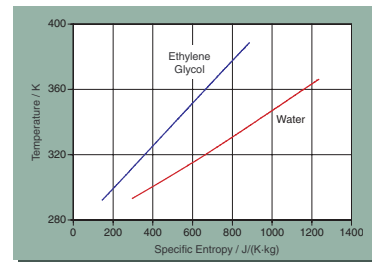# ENTROPY AND TEMPERATURE IN SIMPLE FLUIDS

Fig.2

Fig.3

Here we will study the heating of liquids. Since they can be mixed easily, the entire body can be made to have a single temperature throughout while entropy is absorbed, and this temperature can be measure readily. Solid bodies heat differently, and it is more complicated to determine the changes of entropy and of temperature during heating.

EXPERIMENTS

Here we shall investigate the heating of liquids (water and ethylene glycol) by experiment. The fluids are heated electrically inside a well insulated container. Phase changes (evaporation, vaporization) are not allowed. The amount of entropy added to the fluid is calculated, and the temperature is recorded.

The type of material is responsible for how the temperature increases as a function of the entropy content. With water, for example, the temperature increases linearly with time if the electric power is kept constant. With glycol, the rate of change of temperature decreases with time.

Open WorkBook for this Section

Assignment and Experiment 1

Electric heating of water in a (perfectly) insulated container
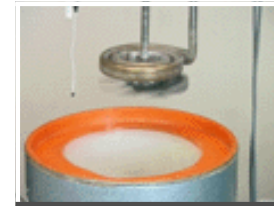
Assignment and Experiment 2

Electric heating of glycol in a (perfectly) insulated container

HEATING WATER IN PERFECTLY INSULATED CONTAINER

Water is heated electrically inside an almost perfectly insulated container. Data of Experiment RAVEL_IV.3-1 are to be used to determine the relation between the specific entropy (entropy per mass) and the temperature of water.

General Assignment

Experiment (RAVEL_IV.3-1) and model (AME_IV.3-1) of the heating of water are to be compared. Time permitting, you should perform the (virtual) experiment, create the model, and determine the warming factor and the entropy capacitance of water.

   a. **Start by investigating the real experiment (RAVEL_IV.3-1).**
   b. **Run through the Project Cycle: perform a first analysis and mental model; perform your own (virtual) experiment; analyze the experiment and create a model.**
   c. **Compare model and experiment. Continue with the Project Cycle if necessary.**

Specific Tasks

   a. **Use the experimental temperature data to calculate the entropy production rate in electric heating (use a spread sheet).**
   b. **Create T–s (temperature – specific entropy) and s–T diagrams for the experiment. Note that they are not linear.**
   c. **Use the s–T diagram and graphically determine the warming factor and the specific entropy capacitance for temperatures 30°C, 40°C… up to 90°C.**
   d. **Compare your k(T) to k = c/T with constant c. Determine an optimal value of c. (c is the specific heat of this fluid.)**
   e. **Create a model which uses the function k = c/T to simulate the heating of water in a perfectly insulated container.**

QUESTIONS & PROBLEMS

Q1. In a simple material such as water, stone, or iron, what does the temperature of a body depend on?

Q2. What is the meaning of "warming factor"? What is the meaning of a small value of the warming factor? Of a large one?

Q3. What is the meaning of specific entropy capacitance?

Q4. Consider granite, iron, and water. Which of these materials has the largest warming factor? Which one has the smallest? Take a temperature of 300 K in each case.

Q5. Qualitatively, what kind of curve represents the temperature–entropy diagram for water? What is the curve in an energy–temperature diagram?

Q6. What is the warming factor of vaporizing water (Section IV.1 and IV.6)? What is the specific entropy capacitance of melting ice? (Does it make sense to define the latter?)

Q7. What does the term "uniform heating" (or "uniform cooling") of a body mean? What does this have to do with how heat spreads through a body? Is this realistic? What does it mean for the model of heating (or cooling)?

Q8. What is the specific temperature coefficient of energy (specific heat) of a body? What is the relation between the (specific) entropy capacitance and the (specific) temperature coefficient of energy of a body?

Q9. What principles did you learn by doing Assignment 1?

———————————————

P1. As determined in Experiment IV.3–1, water has an almost constant specific heat of 4180 J/(K·kg). Set the energy of water to zero at 0°C. Determine the specific energy of water as a function of temperature. Now set the entropy to zero at 0°C. Determine the specific entropy of water as a function of temperature.

P2. The entropy of 10 kg water is 7.06 kJ/K. What is the temperature of the water (if its entropy is zero at 0°C)?

P3. What is the molar entropy of water at 75°C (if the entropy is set to zero at 0°C)? What is the molar energy of water at this temperature?

P4. How much entropy and energy are needed to heat 1000 liters of water from 20°C to 90°C?

HEATING GLYCOL IN PERFECTLY INSULATED CONTAINER

Ethylene glycol is heated electrically inside an almost perfectly insulated container. Data of Experiment RAVEL_IV.3-2 are to be used to determine the relation between the specific entropy (entropy per mass) and the temperature of glycol.

General Assignment

Experiment (RAVEL_IV.3-2) and model (AME_IV.3-2) of the heating of glycol are to be compared. Time permitting, you should perform the (virtual) experiment, create the model, and determine the entropy capacitance (and the specific heat) of glycol.

    a. **Start by investigating the real experiment (RAVEL_IV.3-2).**
    b. **Run through the Project Cycle: perform a first analysis and mental model; perform your own (virtual) experiment; analyze the experiment and create a model.**
    c. **Compare model and experiment. Continue with the Project Cycle if necessary.**

Specific Tasks

    a. **Create T–s (temperature – specific entropy) and s–T diagrams for the experiment from the experimental results.**
    b. **Use the T–s diagram and graphically determine the warming factor, the specific entropy capacitance and the specific heat $c = T \cdot k$ for temperatures 20°C, 40°C… up to 120°C.**
    c. **Show that the specific heat increases (nearly) linearly with temperature ($c = aT + b$).**
    d. **Create a model based on the energy representation which uses the function $c = aT + b$ to simulate the heating of glycol in a perfectly insulated container.**

QUESTIONS & PROBLEMS

Q1. What is the specific temperature coefficient of energy (specific heat) of a body? What is the relation between the (specific) entropy capacitance and the (specific) temperature coefficient of energy of a body?

Q2. Are specific heats of bodies constant in general? Is the specific heat of water constant? Is it constant for glycol?

Q3. What function of temperature is the entropy capacitance of a body whose specific heat is constant?

Q4. If a material has a constant entropy capacitance, what function of temperature is its specific heat (i.e. the temperature coefficient of energy)?

Q5. What is the meaning of the area between the function and the temperature axis in an entropy capacitance – temperature diagram (K_S–T diagram)?

Q6. What is the meaning of the area between the function and the temperature axis in a temperature coefficient of energy versus temperature diagram (C–T diagram)?

Q7. What principles did you learn by doing Assignment 2?

P1. In Experiment IV.3–2, how much entropy and energy have been added to the fluid from a temperature of 40°C to 90°C?

P2. When a body of 100 kg of ethylene glycol is cooled from 80°C to 50°C, by how much does the entropy of this body change?

P3. In the range of temperatures between 310 K and 370 K, a good approximation of the specific heat of pure ethylene glycol is $c(t) = a + bT$ (with $a = 22$ J/(K·kg) and $b = 7.6$ J/(K^2·kg)). What is the specific entropy capacitance in this range (formula and as a diagram)? What is the warming factor in this range?

P4. The T–s relation of glycol determined in Experiment IV.3–2 is nearly linear. What function of temperature should the specific heat be if the T–s relation were to be precisely linear?

P5. Assuming that water and glycol keep their properties when mixed, determine the specific heat of a mixture of 40% water and 60% glycol in the range of 40°C to 100°C.
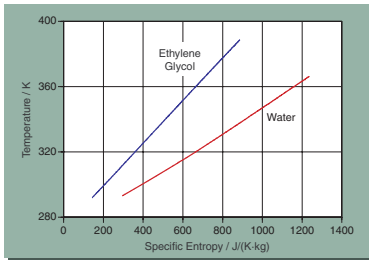
Fig.4

Fig.5

Fig.6

In the simplest type of material (water, sandstone, iron…) the temperature simply depends upon how much entropy it contains. More precisely, the temperature of the material is a function of its specific entropy only. The relation between specific entropy and temperature must be measured, and it can be represented graphically (Fig.4), in a table, or by an analytical approximation.

Measuring the entropy added to a body is simplest in the case of a liquid such as water since it can be stirred during heating which makes the temperature the same throughout the body. The entropy added can be calculated from the energy flowing from the heater (Fig.5): I_S = I_W / T.
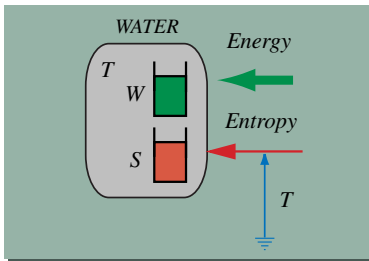
We introduce the specific entropy of a material as the ratio of the entropy it contains and the mass of the body:

$$s = S/m \tag{1}$$

Note that the choice of zero point of the entropy is arbitrary. Mechanical engineers often use 0°C for this purpose.

The most significant feature in the Ts-diagram of a material is the slope of the T(s) curve. It tells us how fast the temperature rises as a function of an increase in entropy. This slope is called the warming factor (Fig.6):

$$\alpha_S = dT/ds \quad \text{or} \quad \dot{T} = \alpha_S \dot{s} \tag{2}$$

If T(s) is a straight line in the Ts-diagram, the temperature can be calculated easily with the help of the heating factor:

$$T = T_{ref} + \alpha_S\left(s - s_{ref}\right) \tag{3}$$
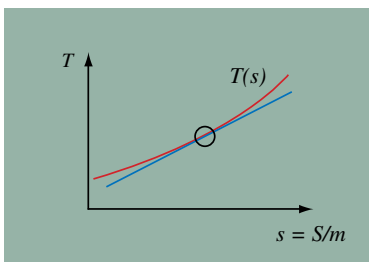
Commonly, the inverse of the heating factor is used to represent the relation between temperature and entropy of a material. The inverse of the heating factor is called the specific entropy capacitance k_S:

$$k_S = ds/dT \quad \text{or} \quad \dot{s} = k_S \dot{T} \qquad \textbf{(4)}$$

The entropy capacitance, K = m·k, has the usual meaning of a capacitance, as in hydraulics, electricity, or rotation. In general, the entropy capacitance depends upon the temperature. The change of entropy of a body can be calculated graphically from the temperature – entropy capacitance diagram (Fig.7).

Fig.7

It is common as well to introduce the specific heat c which is calculated from the specific entropy capacitance by multiplying the latter by the temperature of the material (c = T·k). It allows us to determine directly the change of the energy of the simple materials discussed here:

$$\dot{w} = c\dot{T} \quad \text{where} \quad c = T k_S \qquad \textbf{(5)}$$

The specific heat of materials generally depends upon the temperature of the material. The change of energy of a body can be determined graphically from the specific heat – temperature diagram (Fig.8). Note that the meaning of the specific heat is that of the (specific) temperature coefficient of energy (it is NOT a capacitance!).

Fig.8

Some materials (water, solids at high temperature) have almost constant specific heats. For these materials, entropy and energy can be calculated easily:

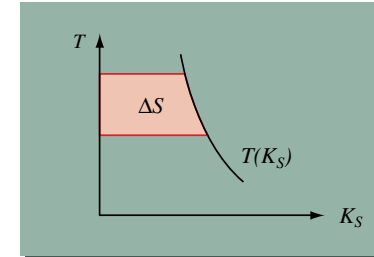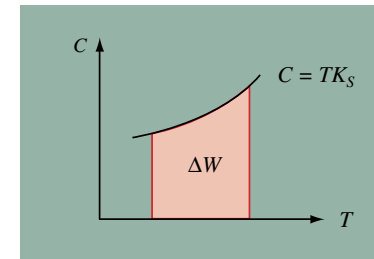$$s = s_{ref} + c\ln\left(T/T_{ref}\right) \quad \text{and} \quad w = w_{ref} + c\left(T - T_{ref}\right) \qquad \textbf{(6)}$$
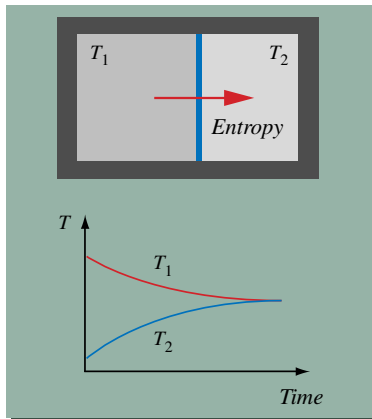
Fig.1

By itself entropy flows from hot to cold places. When two bodies at different temperatures are brought in thermal contact, the hotter cools whereas the colder gets warmer (see Fig.1). Heat transfer uses three mechanisms (Section IV.1):

CONDUCTION: Flow of entropy through bodies as the result of temperature differences;

CONVECTION: Entropy stored in fluids is carried along with these fluids;

RADIATION: Radiation from hot bodies carries entropy.

Here we will study how conduction and convection (at solid-fluid interfaces) combine in the transport of entropy through a series of layers such as those leading from a fluid on one side of a wall to a fluid on the other side of the wall (Fig.2). In each layer the temperature drops (see Fig.3).
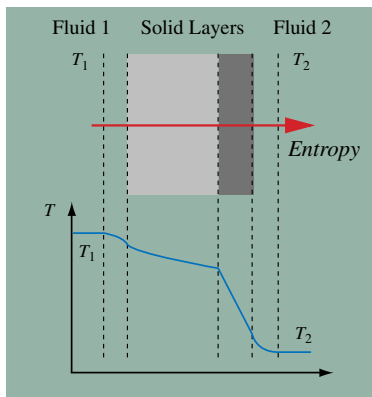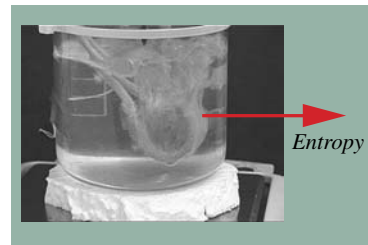
Fig.3

Fig.2

Entropy flows are accompanied by currents of energy. Using the balance of energy alongside the law of balance of entropy in heat transfer we can show that this process is irreversible (see Section IV.5).

EXPERIMENTS

In this section we will study the cooling of liquids (such as water) in sealed containers. This allows us to investigate heat transfer as the result of a temperature difference. Having a closed container prevents evaporation.

Previously we have discussed the relation between the increase of entropy and the rise of temperature of bodies heated at constant volume (see Section IV.3). Combining storage of entropy with entropy transports will let us construct the basic version of dynamical thermal models. By comparing models and experiments we learn how to determine heat transfer coefficients and thermal conductivities.

Open WorkBook for this Section

Assignment and Experiment 1

Cooling of hot water in a thin-walled sealed aluminum container.

Assignment and Experiment 2

Heating of cold water in a thermally insulated sealed container placed in a warm environment.

COOLING OF WATER IN A THIN-WALLED
SEALED CONTAINER

This assignment centers on an experiment in which hot water (or another liquid) is cooled in a thin-walled sealed container such as an aluminum soda can. The water is stirred continuously. The can is polished to make the effect of radiation as small as possible.

General Assignment

Experiment (RAVEL_IV.4-1) and model (AME_IV.4-1) of water cooling in a closed thin-walled aluminum can are to be compared. Time permitting, you should perform the (virtual) experiment, create the model, and determine the heat transfer coefficient from the surface of the container to the air.

> a. **Start by investigating the real experiment (RAVEL_IV.4-1).**
> b. **Run through the Project Cycle: perform a first analysis and mental model; perform your own (virtual) experiment; analyze the experiment and create a model.**
> c. **Compare model and experiment. Continue with the Project Cycle if necessary.**

Specific Tasks

> a. **Compare model and virtual and/or experimental data to determine the entropy conductance for the heat transfer from water to air.**
> b. **Determine the entropy transfer coefficient for the transfer from water to air. Investigate which factors it depends upon (see HML_IV.4-06).**
> c. **Determine the heat transfer coefficient for the transfer from water to air.**

QUESTIONS & PROBLEMS

Q1. What is the importance of the water being mixed constantly, i.e., why should the water be homogeneous or spatially uniform (Figure 1)?

Q2. Which factors determine the rate of change of temperature of the fluid in our experiment at a given moment?

Q3. Why doesn't the water in the real Experiment RAVEL_IV.4–1 reach environmental temperature after a very long time?

Q4. If you had very cold water (from the freezer) in the can and placed it in a warm room, what would happen? Can you sketch the temperature of the water as a function of time?

Q5. Water is heated electrically in a thin-walled metal kettle. Will it boil? Sketch its temperature as a function of time.

Q6. If you double the amount of hot water in a can and adjust the size of the can (with the same ratio of the radius to height) how would the cooling process change?

Q7. Does the cooling process change if you change the ratio of radius to height of the cylindrical can (while the volume stays constant)?

Q8. How does the cooling process change if you blow air across the surface of the container? (The air has always ambient temperature.)

Q9. How does the cooling of a liquid compare to the discharging of an electric capacitor?

Q10. What principles did you learn by doing Assignment 1?

Fig.1

P1. How fast does the temperature of the water in the real Experiment RAVEL IV.4–1 change right at the beginning? At time t = 10,000 s? At time t = 20,000 s? How big is the change of the temperature in the first 20,000 s?

P2. Which factors determine the rate of change of temperature of the fluid in our experiment at a given moment? Determine this quantity formally.

P3. Add heat loss through the lid of the cylindrical container to your dynamical model for Experiment 1.

P4. Express your model in the energy representation if you have used the entropy representation (or vice-versa).

HEATING OF COLD WATER IN AN INSULATED CONTAINER PLACE IN A WARM ENVIRONMENT

This assignment centers on an experiment in which cold water (or another liquid) is heated in a warm environment. The container is a thin-walled metal can surrounded by some insulating material of a certain thickness. The water is stirred continuously.

General Assignment

Experiment (RAVEL_IV.4-2) and model (AME_IV.4-2) of a liquid heated in an insulated can are to be compared. Time permitting, you should perform the (virtual) experiment, create the model, and determine the conductivity of the insulating material. Assume an outside heat transfer coefficient between 7 W/(K·m^2) and 15 W/(K·m^2). The inside heat transfer coefficient is very large.

a. Start by investigating the real experiment (RAVEL_IV.4.-2).
b. Run through the Project Cycle: perform a first analysis and mental model; perform your own (virtual) experiment; analyze the experiment and create a model.
c. Compare model and experiment. Continue with the Project Cycle if necessary.

Specific Tasks

a. Compare model and virtual and/or experimental data to determine the conductivity of the insulating material taking a value of 10 W/(K·m^2) for the outside heat transfer coefficient.
b. Vary the outside heat transfer coefficient and determine its influence upon the value of the conductivity. Having an uncertainty in the heat transfer coefficient, how uncertain is the determination of the conductivity?

QUESTIONS & PROBLEMS

Q1. Would you say the container wall and the insulation act as a pure resistor or should you consider them as storage elements (like the water)?

Q2. How many different transfer layers are there between the water and the air?

Q3. If you represent the transition from water to air by a number of layers, how do you calculate the total thermal resistance of the transition zone?

Q4. Of the mantle, top, and bottom of the container, which one(s) should you include in your determination of the total (energy) conductance for the transfer from air to water?

Q5. If you use the entropy representation of the processes in your model, how do you calculate the entropy flux into the water from the (energy) conductance?

Q6. If the water is not stirred, which temperature do you expect the water to have after a very long time?

Q7. Water and air have the same temperature of 20°C. Now you change the temperature in the room according to half a period of a sine function (going from 20°C to 30°C and back to 20°C in about 8 hours). Sketch the response of the temperature of the water.

Q8. You put a thermometer into hot water. Does it show its temperature immediately without any delay?

Q9. What principles did you learn by doing Assignment 2?

_____

P1. Determine the "half-life" and the time constant in the real Experiment RAVEL IV.4–2. How well does the measured temperature correspond to an exponential function? If the difference is large, does the definition of a time constant make sense?

P2. Consider a cylindrical shell having an inside radius of 4 cm and a thickness of 1 cm made of styrofoam. It has a height of 20 cm. What is its thermal resistance? Can you treat the insulation as having "plane" geometry like a flat sheet (instead of it being a cylindrical shell)?

P3. Which factors determine the rate of change of temperature of the fluid in our experiment at a given moment? Determine this quantity formally.
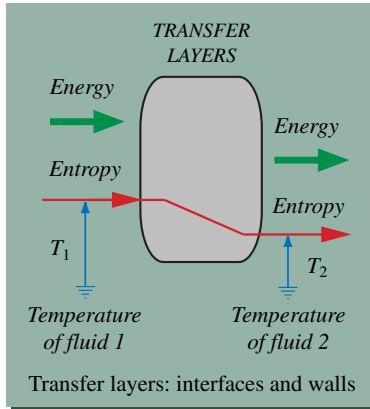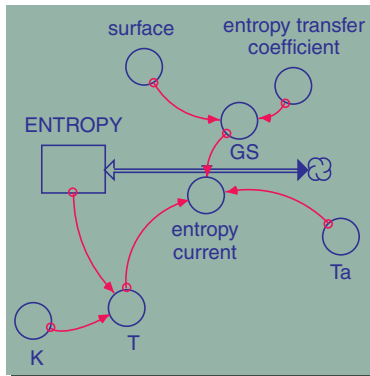
Fig.9

Fig.8

When entropy flows we normally speak of heat transfer. Entropy flow is the result of (1) spontaneous flow through matter due to a temperature difference, (2) convective transport with fluids, or (3) radiation. If one or all of these processes lead to a flow through a series of layers from a hot body to a colder body, we speak of overall entropy flow. The entropy flux I_S is expressed in terms of an entropy conductance G_S and the temperature difference

$$I_S = -G_S \Delta T \tag{2}$$

Usually, the conductance is written in terms the product of an (average) entropy transfer coefficient h_S and the surface A through which the flow takes place:

$$I_S = -h_S A \Delta T \tag{3}$$

The entropy flux through a surface is related to its associated energy current by the temperature T at the surface (Fig.9):

$$I_W = -G_W \Delta T$$
$$G_W = T G_S \tag{5}$$

G_W is called the overall energy conductance. h = T·h_S is called the overall heat transfer coefficient.

Combining this simple expression of an entropy or energy current through a series of layers with the relation of entropy and temperature of a body, and the entropy balance for that body, leads to useful dynamical models for the cooling or heating of uniform bodies (Fig.8). The simplest models of cooling of bodies making use of constant material properties leads to an exponentially decreasing temperature.

The energy conductance of a series of layers (Fig.16) is equal to the inverse of the thermal resistance of this series:

$$G_W = \frac{1}{R_W}$$

which in turn is the sum of the resistances of all the layers:

$$R_W = R_{W,1} + R_{W,2} + R_{W,3} + \ldots \qquad \textbf{(14)}$$

The resistance of a layer depends upon its properties and the type of heat flow. The resistance of a flat conductive layer is

$$R_{W,cond} = \frac{1}{A}\frac{\Delta x}{\lambda} \qquad \textbf{(15)}$$

Fig.16

Lambda is the thermal (energy) conductivity of the material, delta_x is the thickness of the layer, and A is its cross section. Conductive resistances R_cond depend on the geometry of the body conducting heat. Therefore, the expression in Eq.(15) is different for container walls such as a thick cylindrical shell or a spherical shell.

The resistance of a convective interface between fluids and solid bodies (Fig.14) is

$$R_{W,conv} = \frac{1}{Ah_{conv}} \qquad \textbf{(16)}$$

Fig.14

Here, h_conv is the convective heat transfer coefficient which includes all the interesting and difficult details of this type of transfer which depends upon the characteristics of the flow (for example the speed) of the fluid past the solid surface.
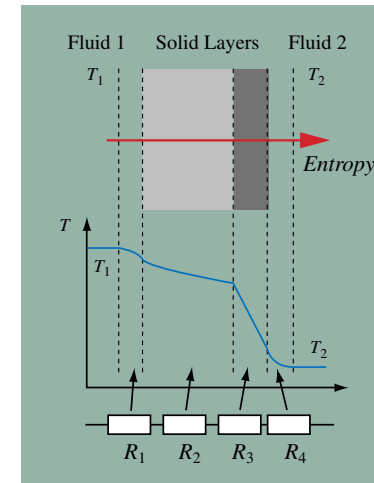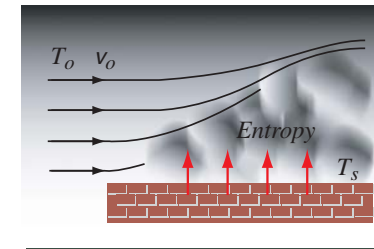
Fig.1

Fig.2

HEAT TRANSFER AND ENTROPY PRODUCTION

When bodies at different temperatures are brought into thermal contact, heat flows from the hotter to the cooler system. This happens if a cold pan is placed on the hot stove, if an immersion heater operates in cold water, or if hot lava flows into the cold ocean water (Fig.1).

We know that if entropy flows from a place of high to a place of low temperature, energy is released (Fig.2). Energy released is used in some follow-up process such as the driving of a heat engine (Section IV.7 and Fig.2 in Section IV.1). If entropy flows through bodies in conduction, or if it flows from a body into a fluid or vice-versa in convective e heat transfer, energy is released as well. However, in general, there is no engine ready to make use of the energy released. (In combinations of some metals, entropy flowing conductively may drive an electric current; this is the principle of the thermoelectric effect). Therefore, the only thing the energy is used for is the production of entropy: heat transfer is irreversible!

Knowing that entropy is produced in entropy transfer immediately tells us that heat transfer through a temperature difference constitutes a "loss" of available power. The energy released in the fall of entropy which is used to produce more entropy cannot be used for a more sensible application. In thermal power plants, "loss" of available power due to heat transfer is the single most important source for the reduction of the efficiency of these engines.
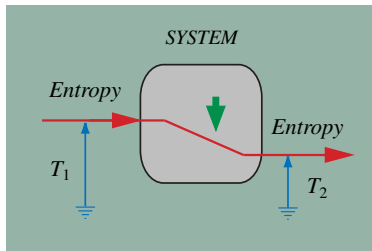
Since much of the technically effected heat transfer is done through the use of heat exchangers, making better heat exchangers is one of the important goals of modern thermal design. Nature has its own agenda in searching for efficient ways to transfer entropy in natural and living systems.

In this section we will study how to model and how to calculate entropy production rates (and total entropy produced) as a consequence of heat transfer. This information is important for models of bodies in thermal contact.

EXPERIMENTS

Here we will study processes which are the result of thermal contact of two or more bodies at different temperatures. Specifically we will work on two experiments: two bodies of water at different temperatures in thermal contact, and a cold copper cylinder immersed in hot water.

The flow of entropy as a consequence of a temperature difference is irreversible. Therefore, in models connecting two or more bodies undergoing change, entropy production as a result of transfer has to be taken into account. In nature and in machines, heat transfer is one of the important sources of irreversibility.

Open WorkBook for this Section

Assignment and Experiment 1

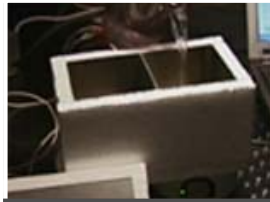Temperature equilibration of two bodies of water in thermal contact

Assignment and Experiment 2

Heating of a cold piece of copper in hot water

TEMPERATURE EQUILIBRATION IN TWO BODIES OF WARM WATER IN THERMAL CONTACT

This assignment centers on an experiment in which two bodies of water are brought in thermal contact. Hot water in one side of a tank, and cold water in the other, are separated by a thin metal wall. The water is stirred continuously.

General Assignment

Experiment (RAVEL_IV.5-1) and model (AME_IV.5-1) of two bodies of water in thermal contact are to be compared. Time permitting, you should perform the (virtual) experiment, create the model, observe the effect of irreversibility and determine the heat transfer coefficient.

a. Start by investigating the real experiment (RAVEL_IV.5-1).
b. Run through the Project Cycles: perform a first analysis and mental model; perform your own (virtual) experiment; analyze the experiment and create a model.
c. Compare model and experiment. Continue with the Project Cycles if necessary.

Specific Tasks

a. Create the model of two bodies of water in thermal contact across a thin metal wall, where the system is insulated from the environment. Use the entropy representation.
a. Compare model and virtual and/or experimental data to determine the entropy conductance for the heat transfer from water to water across the metal wall.
b. Convince yourself that in a model using the entropy representation, entropy production due to heat transfer is necessary to account for the observed temperatures.

QUESTIONS & PROBLEMS

Q1. Two bodies are in thermal contact. Is the entropy absorbed by the colder body equal to the entropy emitted by the hotter one? Is the energy absorbed together with entropy equal to the energy emitted?

Q2. Where does the energy needed to produce entropy in conductive heat transfer come from?

Q3. Consider two tanks connected by a horizontal pipe. If the levels of the liquid become equal after some time, is there entropy production? If so, where does the energy for the production of entropy come from? What would happen if there was no entropy production in the flow of the fluid?

Q4. How is the entropy production rate calculated if entropy flows from a hotter to a colder body?

Q5. It is known that if two identical bodies of water having different temperatures are brought in thermal contact (but are otherwise perfectly insulated from the environment) the resulting final temperature is very nearly the average of the initial temperatures. Why is this so?

Q6. Two bodies of glycol having the same mass but different temperatures are brought in thermal contact (but are otherwise perfectly insulated from the environment). Is the resulting final temperature larger than, equal to, or smaller than the arithmetic average?

Q7. What elements are needed to represent two bodies in thermal contact with the help of process diagrams? How can the process of heat transfer be represented in a system dynamics diagram?

Q8. What principles did you learn by doing Assignment 1?

———————————————

P1. Two identical bodies of water of 1.0 kg mass each having initial temperatures of 80°C and 20°C are brought in thermal contact (but are otherwise perfectly insulated from the environment). How much entropy is produced until the final temperature is reached? Estimate the energy dissipated.

P2. Two equal amounts of glycol of 1.0 kg each having initial temperatures of 120°C and 0°C are brought in thermal contact (but are otherwise perfectly insulated from the environment). How much entropy is produced until the final temperature is reached?

HEATING OF A COPPER CYLINDER
IMMERSED IN HOT WATER

In the experiment upon which the assignment is built, a cold copper cylinder is immersed in hot water inside an almost perfectly insulated container. The water is stirred continuously. The temperatures of copper and water are monitored.

General Assignment

Experiment (RAVEL_IV.5-2) and model (AME_IV.5-2) of a solid body immersed in hot water are to be compared. Time permitting, you should perform the (virtual) experiment, create the model, and determine the specific heat of copper. Assume the heat transfer coefficient from water to copper to be high.

a.   Start by investigating the real experiment (RAVEL_IV.5-2).
b.   Run through the Project Cycles: perform a first analysis and mental model; perform your own (virtual) experiment; analyze the experiment and create a model.
c.   Compare model and experiment. Continue with the Project Cycles if necessary.

Specific Tasks

a.   Compare model and virtual and/or experimental data to determine the entropy capacitance (and the specific heat) and the conductivity of copper.
b.   The temperature of the copper cylinder was measured close to its central axis. How good is a model representing the cylinder as a single uniform body for predicting the temperature a its center? Propose changes in the model if you are not satisfied with the agreement between model and data.

QUESTIONS & PROBLEMS

Q1. Assume Experiment RAVEL_IV.5-2 was performed with a copper cylinder of twice its mass. How would the process of temperature equilibration differ from the one observed?

Q2. Which quantities of the systems in Experiment RAVEL_IV.5-2 are relevant for the final equilibrium temperature?

Q3. Which quantities of the systems in Experiment RAVEL_IV.5-2 determine how fast the equilibrium temperature is attained?

Q4. Consider the copper cylinder immersed in water to have the same temperature as the water. We know that it would be impossible for the water to become hotter and the copper cylinder to get colder spontaneously. What is the reason for this? Is it because there is no energy that can be released to drive the process, or is it because in the reverse process entropy would have to be destroyed?

Q5. Copper is a very good thermal conductor. Still, the temperature of the copper cylinder could not be the same at every point inside during the experiment. How could you model the process of temperature equilibration between water and copper taking account of the fact that the copper cylinder is inhomogeneous during the heating?

Q6. What principles did you learn by doing Assignment 2?

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P1. A block of aluminum having a mass of 0.900 kg with an initial temperature of 200°C is immersed in 1.5 kg of water having an initial temperature of 20°C. The water is inside a container having an energy capacitance (temperature coefficient of energy) of 200 J/K. The container is well insulated to the environment. The final temperature reached by the water is measured to be 40°C. What is the specific heat of aluminum? (All specific heats are assumed to be independent of temperature.)

P2. Using the graph of temperatures versus time of Experiment RAVEL_IV.5-2 determine the time constant of the system composed of water and copper. Knowing the energy capacitances of the bodies (water and copper) determine the value of the thermal resistance for the transfer from water to copper. (Assume water and copper cylinder to be homogenous.) Explain this value in terms of the properties of the transfer layers.

P3. For two bodies in thermal contact having constant energy capacitances and constant energy transfer coefficients, determine the equations describing the process of temperature equilibration. Show that the solutions are exponential functions and derive the expression for the time constant of the process.

Fig.3

Heat transfer, i.e., conductive entropy transfer, is irreversible. Entropy is produced always if entropy flows from a hotter to a colder place. This effect has to be taken into account in dynamical models involving more than one body, i.e., as soon as we have two or more bodies in thermal contact (Fig.3) whose thermal processes we want to understand.

Since entropy production is a consequence of the heat transfer, and the transfer takes places through transfer layers (Fig.4), we have to modify our model of a thermal resistive element (or elements). The process diagram of a thermal resistor must include a source due to entropy production (Fig.5).

Fig.4

Fig.5

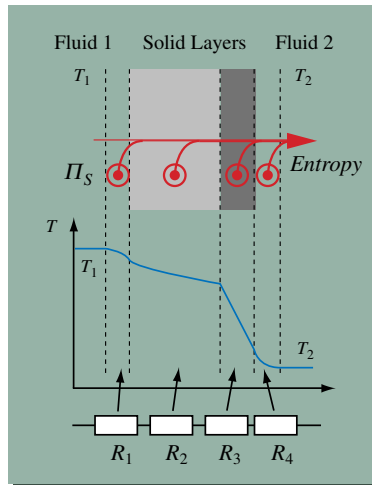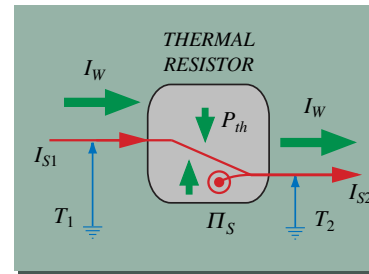The entropy production rate in a thermal resistive element is again calculated from the dissipation rate. The energy dissipated is the energy released by the fall of entropy from T_1 to T_2 (Fig.5). The thermal power of the entropy current entering the system is $P\_{th} = (T\_1 – T\_2) \cdot I\_{S1}$. Since the dissipation takes place at the lower temperature T_2, the entropy production rate is

$$\Pi_S = \frac{T_1 - T_2}{T_2} I_{S1} \qquad \textbf{(1)}$$

A possible representation of entropy production due to entropy transfer in a system dynamics model is the following (Fig.6). The balance of entropy of the two bodies exchanging heat involves two symbols for the respective entropies. Without dissipation, the exchanged could be represented by a direct flow between the storage elements. Now we add a third stock as a symbol for the entropy of the transfer layer(s) between the bodies. Since the transfer layer is a resistor, it does not store entropy. Therefore, the flow out of this element equals the sum of the flow in and the entropy production rate in the element:
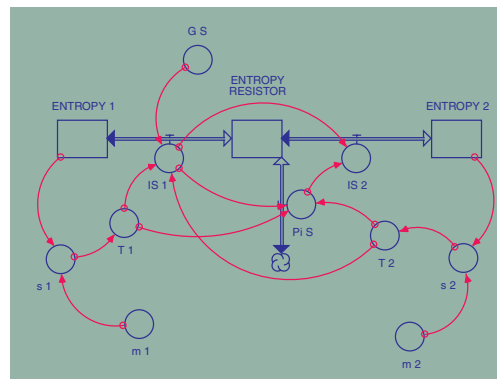
$$I_{S2} = I_{S1} + \Pi_S \qquad \textbf{(2)}$$

Fig.6

Heat does not only make bodies warmer, or air expand, it is also the necessary ingredient for phase changes: we need entropy to make ice melt (Fig.1) or water vaporize (Fig.2).
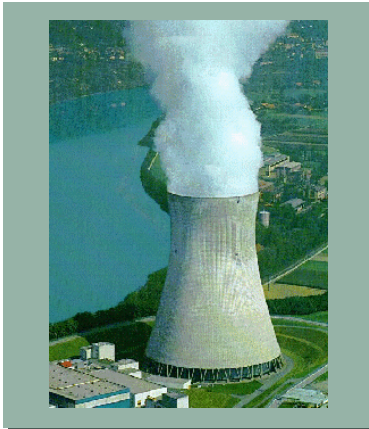
Fig.2

If entropy is needed to melt a substance, the resulting system, i.e., the mixture of solid and liquid (or the liquid alone), must contain more entropy then the purely solid body. Moreover, we know from experience that the temperature of a melting substance does not change: the entropy is used purely to make the phase change happen. The same is true for vaporization of a liquid (effected at constant pressure). The temperature of fusion (or the temperature of vaporization) is unique for the substance and its conditions.

If melting (or vaporization) of a substance such as ice/water increases its entropy but does not change the temperature, the TS-diagram of the substance looks like the one in Fig.3.
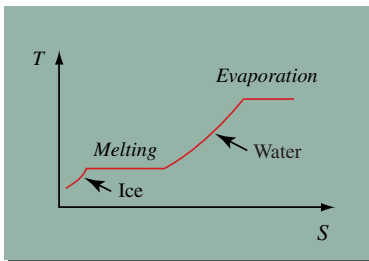
In this section we will study by how much the entropy of a substance must change if we want to change its phase. In the case of vaporization we shall also look at the dependence of the vaporization temperature upon the pressure of the fluid. We will learn how to incorporate this information in dynamical models of thermal processes.

Fig.3

Entropy can make materials melt or vaporize. If a vapor condenses or a liquid fuses, they give off entropy. The change of entropy of the mixture of solid/liquid or liquid/steam is proportional to the amount of substance melted or vaporized.

Here we will study two experiments showing different aspects of heating, melting, and vaporization. In the first, water is boiled and vaporized, and the mass of steam generated is determined. In the second, ice in a test tube is inserted into hot water and melted.

Open WorkBook for this Section

Assignment and Experiment 1

Boiling and vaporizing water in an insulated container (open at the top) placed on a scale

Assignment and Experiment 2

Melting and warming of ice/water in a test tube placed in hot water in a sealed and insulated container

VAPORIZING WATER IN AN INSULATED OPEN CONTAINER PLACED ON A SCALE

Water is boiled in an insulated container open at the top (Experiment RAVEL_IV.6-1). To measure the rate at which water is converted to steam, the container is placed on an electronic scale.
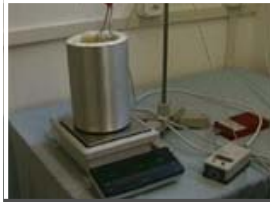
General Assignment

Experiment (RAVEL_IV.6-1) and model (AME_IV.6-1) of the vaporization of water are to be compared. Time permitting, you should perform the (virtual) experiment, create the model, and determine the entropy of vaporization of water.

a. **Start by investigating the real experiment (RAVEL_IV.6-1).**
b. **Run through the Project Cycle: perform a first analysis and mental model; perform your own (virtual) experiment; analyze the experiment and create a model.**
c. **Compare model and experiment. Continue with the Project Cycle if necessary.**

Specific Tasks

a. **Do the following on paper (without the use of the dynamical model). Determine the rate of change of mass of the water, and the mass flux of steam. From the electric power of the heater and the temperature of vaporization, determine the rate of production of entropy. Determine the specific and the molar entropy of vaporization of water (at roughly 100°C). Calculate the latent heat of vaporization.**
b. **Create the dynamical model (based on the entropy representation) to go with his experiment and determine the entropy of vaporization of water with its help.**

QUESTIONS & PROBLEMS

Q1. What is needed to vaporize water?

Q2. Which quantities stay constant and which change during the vaporization of water (at constant pressure)?

Q3. Which laws of balance are of interest in the case of vaporization of a body of water?

Q4. Water is boiled and vaporized in an open glass container with the help of an immersion heater. Look at the water only. What is the law of balance of entropy for this system?

Q5. Does the specific entropy of water which is boiling and vaporizing change? Does its total entropy change?

Q6. Water is vaporized using an immersion heater operating at constant electric power. Should we expect the amount of water to decrease linearly? What about the amount of vapor?

Q7. How do you determine the rate of production of entropy by the electric heater during Experiment RAVEL_IV.6-1?

Q8. Can you be confident that all the entropy produced in the electric heater in Experiment RAVEL_IV.6-1 goes toward the vaporization of water? Does all the energy released by the heater end up in the steam?

Q9. Is the process of vaporization (or of condensation) reversible?

Q10. Hot lava flows into cold ocean water, and some water vaporizes. Is the process of vaporizing water with hot lava reversible?

Q11. What principles did you learn by doing Assignment 1?

P1. What is the rate of change of the mass of water in Experiment RAVEL IV.6-1? What is the rate of change of amount of substance?

P2. What is the change of the entropy of the steam generated in Experiment RAVEL_IV.6-1 for the duration of the experiment? What is the change of the entropy of the water during this time span?

MELTING AND WARMING OF ICE/WATER IN A TEST TUBE PLACED IN HOT WATER

In Experiment RAVEL_IV.6-02, ice at –18°C in a test tube is placed in hot water inside a sealed and insulated container. The water is mixed constantly. The ice is first warmed, then melted, whereupon the water in the test tube is warmed until its temperature is the same as that of the surrounding.

General Assignment

Experiment (RAVEL_IV.6-2) and model (AME_IV.6-2) of ice warming and melting, and water warming in a test tube are to be compared.

   a. **Start by investigating the real experiment (RAVEL_IV.6.-2).**
   b. **Run through the Project Cycle: perform a first analysis and mental model; perform your own (virtual) experiment; analyze the experiment and create a model.**
   c. **Compare model and experiment. Continue with the Project Cycle if necessary.**

Specific Tasks

   a. **Sketch and then determine quantitatively the s–T diagram of ice, ice/water, and water. Get the specific heats of ice and water and the specific heat of melting from tables. Set the entropy equal zero at –20°C.**
   b. **Determine the inverse function (T–s) of the one calculated in a. (Alternatively, use the result provided in T-s_Data_Ice-Water.XLS.)**
   c. **Create a model of the warming and melting of ice in the test tube treating the material as a homogeneous body. Assume the water in the container to be mixed at all times. The container is well insulated.**

QUESTIONS & PROBLEMS

Q1. What happens with entropy during the fusion of ice? What happens when ice is melting?

Q2. Is the process running in Experiment RAVEL_IV.6-2 reversible? Is the melting of ice by itself reversible?

Q3. Which quantities do you need to know to calculate the rate at which entropy is needed during the melting of ice?

Q4. Why are the amounts of entropy and energy absorbed during the melting of ice proportional? What is the factor of proportionality?

Q5. Assume you have some tubes full of ice inside a tank with glycol, all of this at a temperature of about – 20ºC. Now the glycol is heated slowly. How do you expect the temperature of glycol to evolve?

Q6. What principles did you learn by doing Assignment 2?

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P1. How much entropy and energy are emitted by 1 kg water which is cooled from 20ºC to – 20ºC?

P2. How much ice of 0ºC is needed to make water of 8ºC if it is mixed with 1 kg of water at 25ºC? How much entropy is produced as the result of the process? Water and ice are inside a well insulated box.

P3. In Experiment RAVEL_IV.6-2, what should be the final temperature of water and ice-in-test-tube when equilibrium is reached?
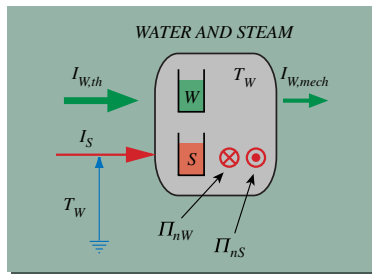
WATER AND STEAM

Fig.4

When a substance is melted or vaporized, several processes take place at the same time (Fig.4). First of all, entropy is needed for the phase change which means that entropy and energy are added to the substance upon which the stored quantities of entropy and energy (of the system composed of both phases) increases. At the same time some energy may be emitted (or absorbed) as the consequence of a volume change of the systems (this energy flow is the result of a mechanical process). This is important in vaporization, but much less so in melting. Therefore the laws of balance of entropy and energy for the systems are

$$\dot{S} = I_S \tag{1}$$

$$\dot{W} = I_{W,th} + I_{W,mech} \tag{2}$$

At the same time, a chemical process takes place: one of the phases of the substance disappears, whereas the other is produced. The laws of balance of amount of substance are

$$\dot{n}_1 = \Pi_{n1} \quad , \quad \dot{n}_2 = \Pi_{n2} \quad , \quad \Pi_{n1} = -\Pi_{n2} \tag{3}$$

The constitutive law of a phase change is simple. It expresses the relation between the rate at which entropy is needed and the rate at which one of the phases is produced. The two are proportional and simply depend on the substance and its conditions:

$$\dot{S} = \lambda_m \dot{n} \tag{4}$$

lambda_m is called the molar latent entropy of fusion or of vaporization of the substance undergoing phase change. The specific latent entropy is calculated as follows: lambda = lambda_m/M_o, where M_o is the molar mass of the substance.

The energy flow accompanying the entropy flow in the heating during phase change is calculated simply by

$$I_{W,th} = q\dot{m} \quad , \quad q = T\lambda \qquad (5)$$

This is so since the temperature during phase change is constant.

Phase change temperatures and latent entropies depend upon the circumstances of the phase change. The most important factor is the pressure. This is particularly important in the case of vaporization.

The temperature of vaporization is sensitive to the pressure of the fluid. There is a unique pressure-temperature relation for the phase change which is called the vapor pressure of the substance (Fig.5 for water). The vapor pressure of water at 100°C is 1 bar. The Ts-diagram therefore depends upon the pressure of the fluid as well (Fig.6 for water).
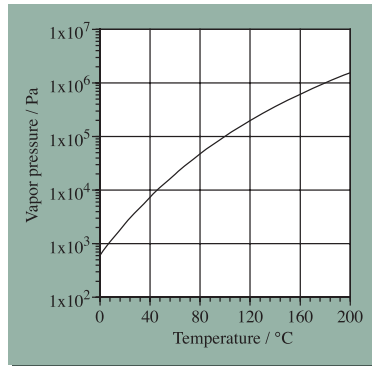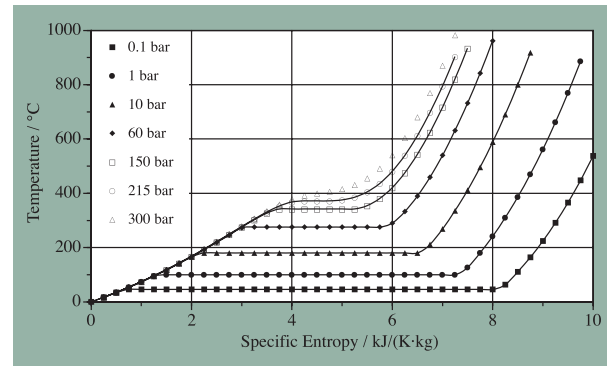
Fig.5

Fig.6

Assignment 1

THERMAL PHENOMENA

Q1. The temperature does not change even though we add heat.

Q2. Horizontal line from right to left at 100°C.

Q3. Iron: Top right to bottom left. Environment: horizontal from left to right.

Q4. Entropy is produced, and it leaves the heater.

Q5. Water and vapor: $dS/dt = \Pi\_S - I\_S\_loss$ (through wall). Water only: $dS/dt = \Pi\_S - I\_S\_loss - I\_S\_vapor$.

Q6. Storage, flow, and production.

Q7. $S\_e = \Delta S - S\_prod$

Q8. Heating: adding entropy to a body. Cooling: withdrawing entropy from a body. These terms are not the same as getting warmer or colder.

Q9. Reversible: vertical line downward. Irreversible: Line from top left to bottom right (entropy is produced!).

Q10. Yes, if you compress it and cool it at the same time.

Q11. TS diagrams are the same. Law of balance is different (with fire: $dS/dt = \Pi\_S$; with heating from outside: $dS/dt = I\_S$).

Q12. In your own words…

P1. Law of balance of entropy: $dS/dt = I\_S + \Pi\_S$. Therefore: $\Pi\_S = dS/dt - I\_S = 0.10$ W/K $-$ (-0.060 W/K) $= 0.16$ W/K. (In this calculation we assume that sun light does not have any entropy. Actually, there is a small rate of absorption of entropy as a result of the absorption of sun light.)

P2. Draw the rate of change in a $dS/dt - t$ diagram and integrate. Result: $\Delta S = 0.075$ W/K $\cdot$ 20 s $= 1.5$ J/K.

Assignment 2

Q1. See Figure 7 of the Summary of this section.

Q2. Dissipation rate is the name for the rate at which energy is bound (used) for the production of entropy. It is equal to the product of the rate of production of entropy and the temperature at which dissipation takes place (Summary, Equation 3).

Q3. $D = U \cdot I\_Q/T$

Q4. By integrating the dissipation rate over time (area between curve of function and time axis in a Π_S – t diagram).

Q5. Yes, if there is no loss of entropy to the environment (no heat loss) and no other source of entropy production in the water. (There should be a very small amount of extra entropy production due to the mechanical friction with the mixer.)

Q6. Mechanical power divided by the (Kelvin) temperature of your hands.

Q7. It must be equal to the rate of absorption of energy (from radiation) divided by the temperature of the absorber. (Actually, this is only true if the light of the Sun does not bring any entropy which is absorbed with the radiation. Solar light carries about 5% of the entropy which is produced when it is absorbed in a body on Earth at a temperature of about 300 K).

Q8. If hotness did not have an absolute zero, we could have negative temperatures. With negative temperatures, the dissipation equation would lead to negative production rates of entropy (entropy destruction, which cannot take place!).

Q9. In your own words…

P1. Roughly $200/(273+40) \cdot 100$ J/K = 64 J/K (draw $\Pi\_S = P\_el/T$ as a function of time and integrate!).

P2. It is not equal. The entropy production rate decreases with increasing temperature (at constant power).

P3. $P\_grav = P\_el / 0.82 = 245$ MW. Therefore, the dissipation rate is 45 MW. The temperature of dissipation is close to the environmental temperature of about 300 K. Therefore, the entropy production rate is 0.15 MW/K.

HOTNESS AND TEMPERATURE

Assignment 1

Q1. It measures its own temperature. Only if the body has the same temperature can we use the reading for the quantity we are seeking.

Q2. It takes entropy from the (hotter) body until it has the same temperature as the body. The thermometer changes its temperature according to its entropy change (depending upon the properties of the thermometer).

Q3. Only in the range of 4°C to about 100°C.

Q4. Measure voltage and electric current. If the resistive thermometer is a wire, use the formula for the resistance of a wire.

Q5. Put both in the same well mixed fluid for a time long enough for steady-state conditions to be applicable.

Q6. It takes time to reflect the change. In the simplest case, it responds like a fluid container to charging or discharging.

Q7. The coefficient multiplying the change of temperature to yield the relative change of the resistance.

Q8. No.

Q9. Yes, it could mean that. As long as we do not know what a "true" temperature is supposed to be, and how mercury behaves, we cannot be sure about the properties of the mercury scale.

Q10. In your own words…

P1. The linear coefficient is 0.0109 K^−1; the quadratic coefficient is 0.00207 K^−2. The resistance at 30°C is 56.6 Ω.

P2. $\rho_0 = 1.78 \cdot 10^{-8}$ $\Omega \cdot$m; $R_0 = \rho_0 \cdot l / A = 56.7$ Ω; $R = R_0 \cdot (1 + \alpha \Delta T) = 96.4$ Ω (linear approximation only).

P3. 17 s.

Assignment 2

Q1. The pressure of the gas is used to determine the temperature.

Q2. The pressure of the air in the bulb equals the ambient air pressure plus the hydrostatic pressure of the difference of the mercury column in the pipes of the pressure gauge. Therefore, the ambient pressure plays a role.

Q3. No, you cannot. You have investigated only one substance (air). Furthermore, you have assumed that air retains the pressure – temperature relation found at high temperatures down to low temperatures to infer that there is a lowest possible temperature. In fact, all you have shown is that there would be a lowest temperature for air if it retained the ideal gas behavior at all temperatures.

Q4. No. Long before a temperature of 0 K is reached, the different gases in the air liquefy, making the ideal gas relation invalid.

Q5. It would still be a straight line with twice the slope of the one found in the experiment.

Q6. You need the same amount of substance to have the same pressure at the same temperature in the same volume. If there was one mole of air, you would need one mole of helium.

Q7. In your own words…

P1. With an error in the ambient air pressure, the pressure of the air in the bulb would be shifted by the same amount for every reading. We would get a linear fit of the data which is shifted up or down. Naturally, this linear function would intersect the temperature axis at a different value.

P2. Use the ideal gas law to determine the pressure: $p = n*R*T / V = 0.010 \cdot 8.314 * (0 + 273) / 0.1 \cdot 10^{-3}$ Pa $= 2.27 \cdot 10^5$ Pa.

ENTROPY AND TEMPERATURE IN SIMPLE FLUIDS

Assignment 1

Q1. The temperature of a simple material depends upon its specific entropy only.

Q2. The warming factor tells us how easy it is to change the temperature of a body. A small value of the warming factor means that we need a lot of entropy for a given temperature change; a large warming factor means that we only need a little bit of entropy. The warming factor is found as the slope of the tangent to the temperature–specific entropy curve.

Q3. The specific entropy capacitance of a material is the inverse of the warming factor.

Q4. Of granite, iron, and water, iron has the largest warming factor, and water has the smallest.

Q5. The T–s diagram of water is a curve that increases faster at higher temperatures. The temperature–energy curve is a straight line.

Q6. The warming factor of vaporizing water is zero. The entropy capacitance of melting ice cannot be defined (as the inverse of the warming factor, it would be infinitely large).

Q7. Uniform heating means that the temperature of the body is taken to be spatially uniform during the process. This can be achieved easily for liquids or gases (if they are stirred). In the case of a solid, however, uniform heating would require that entropy can spread through the body without a temperature difference (we would need infinitely high conductivity). Dynamical models of uniform bodies are particularly simple.

Q8. The specific heat of a material tells us by how much the energy of the material changes if we change its temperature by one K. It is equal to the specific entropy capacitance multiplied by the temperature of the material.

Q9. In your own words…

P1. $w = c \cdot \theta$ (θ is the celsius Temperature); $s = c \cdot \ln(T/273)$.

P2. $7060 = 10 \cdot 4200 \cdot \ln(T/273)$, therefore $T = 323$ K.

P3. $S/n = M_o \cdot c \cdot \ln((75+273)/273) = 0.018 \cdot 4200 \cdot 0.243$ J/(K·mole) $= 18.3$ J/(K·mole). $W/n = M_o \cdot c \cdot \theta = 0.018 \cdot 4200 \cdot 75$ J/mole $= 5.67$ kJ/mole.

P4. $\Delta S = m \cdot c \cdot \ln(T2/T1) = 1000 \cdot 4200 \cdot \ln(363/293)$ J/K $= 0.90$ MJ/K. $\Delta W = m \cdot c \cdot (T2 - T1) = 294$ MJ.

Assignment 2

Q1. The specific heat of a body is the material property which relates the rate of change of temperature of the body to its rate of change of energy. The specific temperature coefficient of energy (specific heat) is obtained by multiplying the specific entropy capacitance by the temperature of the material.

Q2. In general, specific heats (like specific entropy capacitances) depend upon the temperature of the material. The specific heat of liquid water is almost constant. It is not constant for glycol (it increases with temperature).

Q3. If the specific heat is constant, the entropy capacitance is inversely proportional to the temperature of the body ($K\_S = m \cdot c / T$).

Q4. For constant entropy capacitance, the specific heat is proportional to the temperature of the material ($c = T \cdot k\_s$).

Q5. This quantity corresponds to the change of entropy of the body.

Q6. This quantity corresponds to the change of energy of the body.

Q7. In your own words…

_____

P1. $\Delta S = K\_S \cdot \Delta T = 1.398 \cdot 7.67 \cdot 50$ J/K = 536 J/K; $\Delta W = 0.5 \cdot (C(40) + C(90)) \cdot \Delta T = 1.398 \cdot 0.5 \cdot (2385 + 2757) \cdot 50$ J = 180 kJ.

P2. $\Delta S = K\_S \cdot \Delta T = 100 \cdot 7.67 \cdot (50-80)$ J/K = − 23 kJ/K.

P3. $k\_S(T) = c(T)/T = (a + bT)/T = a/T + b$; $a\_S(T) = T/(a + bT)$.

P4. If $T = a + b \cdot S$, then $k\_S = 1/b$ (i.e., it is constant), and $c(T) = 1/b \cdot T$.

P5. $c\_{mixture} = 0.40 \cdot c\_{water} + 0.60 \cdot c\_{glycol}$; $c\_{water} = 4200$ J/(K·kg); c_glycol can be taken from Problem P3. The mixture does not have a constant specific heat (just like glycol).

SECTION IV.4

ENTROPY AND ENERGY IN HEAT TRANSFER

Assignment 1

Q1. The reason is simply that we want to have a body with only a single value of the temperature throughout. Homogeneous bodies can be modeled easily in terms of a single stock in a system dynamics model. (Moreover, it is easy to set up a theory of reversible processes of spatially uniform bodies. Homogenous bodies are like thermal "superconductors".)

Q2. The rate of change of temperature depends upon the rate of change of entropy of the water (which depends upon the net flow and the production rate of entropy in the water) and on the entropy capacitance of the body. The entropy flow in turn depends upon the difference of temperatures of fluid and environment, the surface area and the overall entropy transfer coefficient. The entropy capacitance depends upon the specific entropy capacitance of the material and the mass of the body. As to the overall entropy transfer coefficient, it depends upon the nature of transfer from the water to the can, through the wall of the can, and from there to the ambient air. The most important factor of the three is the last.

Q3. The constant mixing of the water produces entropy. In steady-state, there has to be an equal entropy flow out of the water for which a temperature difference is needed.

Q4. Heat would flow from the warm environment into the cold liquid in the can. The temperature of the liquid rises fast at first, and more slowly later on (the function looks like the mirrored version of the one you know from Experiment IV.1).

Q5. The temperature of the water will rise fast at first, and more slowly later on because the flow of heat out of the water will increase with increasing temperature difference. Whether or not the water will boil depends on the rate of electric heating (as compared to the heat loss when the water is hot). There may be a steady-state at a temperature below the boiling temperature.

Q6. The look of the temperature curve will not change. However, the process will be slower (the time constant is larger) because the amount of liquid is doubled (giving us twice the entropy stored) and the surface area (responsible for entropy loss) increases by a factor smaller than 2.

Q7. The form of the cooling curve does not change. However, since the surface area (responsible for entropy loss) changes with the shape of the can, the rate of heat loss changes. For minimal surface area the cooling is the slowest.

Q8. With increasing wind speed the effect of convective cooling at the surface of the can increases, making the cooling faster.

Q9. Fundamentally, they are similar. There may be differences in detail depending on the form of the capacitance laws and the laws for the flows of charge or entropy.

Q10. In your own words…

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P1. The rate of change of temperature is determined graphically by taking the slope of the tangent to the data curve. We get $-7.7 \cdot 10^{-3}$ K/s at t = 0 s; $- 2.0 \cdot 10^{-3}$ K/s at 1000 s; $- 6.2 \cdot 10^{-4}$ K/s at 20000 s. The change of temperature is $- 53.5$ K in the first 20000 s.

P2. The rate of change of temperature is determined by the rate of change of entropy of the body and the entropy capacitance. The rate of change of entropy is equal to the (net) flow of entropy plus the rate of production of entropy. The (net) flow is given by the product of conductance and temperature difference, whereas the rate of production of entropy is determined by the dissipation rate and the temperature of the fluid. The entropy capacitance is that of the body of water.

P3. Add a flow to the stock for the entropy of the water. Make the flow dependent upon the difference of temperatures of water and environment, upon the surface area of the lid, and upon an entropy transfer coefficient for the transfer from water to air through the lid (if the lid is insulated, this transfer coefficient is small).

P4. The entropy and the energy representations of the model must lead to the same results.

Assignment 2

Q1. Probably the model of a pure resistor is fine. The entropy capacitances of the thin metal wall and the insulation are expected to be small compared to that of the water.

Q2. From water to metal wall (inside); through the metal; through the insulation; from the surface of the insulation to the air. (There may be a transfer layer from the metal wall to the insulation.)

Q3. The total thermal resistance (the energy flow resistance) is calculated as the sum of all single resistance values.

Q4. The mantle is the most important part. The top has a small area and is heavily insulated with cotton, and can probably be neglected. The bottom rests upon the (plastic) surface of the magnetic mixer which should be a little warmer than the environment. Even though the area of the bottom of the container is small compared to that of the mantle, we might add the effect of heat transfer through the bottom just to see if it makes a big difference.

Q5. The model yields the energy flux through the series of layers. The energy flux divided by the temperature of the water equals the entropy flux entering the water.

Q6. The water should attain the same temperature as the environment. However, this assumes, that the temperature of the environment does not change (or at least not very fast; you should try to investigate the effect of oscillations in the ambient temperature).

Q7. If we neglect the effect of entropy added due to mixing, the temperature of the water will rise to about 25.6°C with a big delay (maximum at about 22000 s) and then fall again to about 24.9°C at the end of the half period. (Results from the dynamical model with an outside heat transfer coefficient of 12 W/(K·m^2) and a conductivity of the insulation of 0.03 W/(K·m).)

Q8. There will be a delay whose size depends upon the time constant of the thermometer in its new environment. This time constant is determined by the capacitance of the thermometer and the transfer coefficient between thermometer and water.

Q9. In your own words…

P1. The time constant of the system can be obtained by determining how

long it would take for the water to reach the final steady temperature if it continued to change at its initial rate. We get about 17000 s. If we determine how long it takes until the temperature difference between initial temperature and final temperature has reached 36%, we obtain roughly 19000 s. (The half-life is about 12000 s.) On the other hand, the function T(t) = 273 + 1.5 + 25*(1 - EXP(-t/18000)) approximates the measured temperature relatively well. This means that the time constant is around 18000 s.

P2. $R\_W = 1/(2 \cdot \pi \cdot 0.20 \cdot 0.036) \cdot \ln(0.050/0.040) = 4.9$ K/W. Flat geometry: $R\_W = 0.010/(2 \cdot \pi \cdot 0.045 \cdot 0.20 \cdot 0.036)$ K/W = 4.9 K/W (the difference is very small).

P3. The rate of change of temperature is determined by the rate of change of entropy of the body and the entropy capacitance. The rate of change of entropy is equal to the (net) flow of entropy plus the rate of production of entropy. The (net) flow is given by the product of conductance and temperature difference, whereas the rate of production of entropy is determined by the dissipation rate and the temperature of the fluid. The entropy capacitance is that of the body of water.

Part IV: THE DYNAMICS OF HEAT

SECTION IV.5

HEAT TRANSFER AND ENTROPY PRODUCTION

Assignment 1

Q1. The entropy absorbed by the cooler body is larger than the entropy emitted by the hotter, since entropy is produced in the process. The energy transferred out of the hotter body is equal to the energy transferred into the colder one (energy is conserved, and there are no other energy transfers than the ones associated with heat transfer).

Q2. The energy needed for the process of entropy production is the energy released in the fall of entropy from a high to a lower temperature.

Q3. There is entropy production because of fluid friction. The energy for entropy production is the energy released in the flow process in the pipe (pressure difference because of friction). If there were no entropy production, the fluid should oscillate back and forth forever.

Q4. The entropy production rate equals the rate at which energy is released (which equals the product of temperature difference and entropy current) divided by the lower of the two temperatures.

Q5. This is the result of the particular form of the entropy capacitance of water (which is inversely proportional to the temperature of the water). Because of this the specific heat of water is constant (independent of temperature).

Q6. The final temperature is higher than the arithmetic mean, because the specific heat of glycol is higher for higher temperatures. Expressed in terms of entropy: the entropy capacitance is almost constant. Without entropy production we would expect exactly the arithmetic mean for the final temperature. Since we have entropy production we will get a higher temperature.

Q7. In process diagrams, we need two elements for the uniform bodies and one for the thermal resistor between them which represents the transfer element. In a system dynamics diagram we need two stocks for the entropy stored in the bodies and one stock for the entropy of the resistor (which we model to be a node which does not store entropy).

Alternatively, if we use the energy representation, we just need two stocks for the energies of the bodies and a flow symbol connecting the stocks.

Q8. In your own words…

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P1. $S\_gen = DS$; $DS = DS1 + DS2 = m1 \cdot c \cdot \ln(Tf/T10) + m2 \cdot c \cdot \ln(Tf/T20) = 36$ J/K. The energy dissipated can be estimated from $W\_diss =$

T_diss_av·S_gen = 308·36 J = 11.2 kJ. Alternatively, we can use the system dynamics model of the process and integrate the entropy production rate and the dissipation rate over time (the energy dissipated turns out to be equal to 11.0 kJ).

P2. The entropy capacitance is constant. $S\_gen = DS$; $DS = DS1 + DS2 = m1·k\_S·(Tf − T10) + m1·k\_S·(Tf − T20)$ with $k\_S = 7.6$ J/(K^2·kg). The final temperature Tf is obtained from the balance of energy (remember that the specific heat of glycol changes linearly with temperature): $m1·0.5·k\_S·(T10 + Tf)·(Tf − T10) = −m2·0.5·k\_S·(T20 + Tf)·(Tf − T20)$. Therefore: $Tf = SQRT(0.5·(T10^2 + T20^2)) = 338.4$ K (= 65.4°C > 60°C). Therefore: $S\_gen = 81.5$ J/K.

Assignment 2

Q1. The temperature reached at the end (equilibrium temperature) will be lower, and the process will run more slowly.

Q2. The masses and specific heats of the bodies in thermal contact (and, naturally, the initial temperatures). These are the copper cylinder, water, and the glass container.

Q3. The surface areas of the copper cylinder (for the contact of water and copper) and the surface area of the glass container (for the contact of water and glass), the entropy transfer coefficients for the interfaces between water and glass and water and copper, and the entropy conductance of copper.

Q4. We need energy to pump entropy from one body to another. This energy has to be supplied from outside as in all other similar systems as well (e.g., water at rest at equal levels in two communicating cylinders). It is true that entropy would have to be destroyed if the process occurred spontaneously. This is the case as well in other processes which do not happen spontaneously. So we may look at the lack of available energy as the "real reason" for such a process not to occur.

Q5. The copper cylinder would have to be modeled as consisting of (many) connected parts where each part is modeled as a uniform body. This is called a finite element model of the body.

Q6. In your own words…

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P1. Use the balance of energy: $m\_W \cdot c\_W \cdot (Tf - Ti\_W) + C\_cont \cdot (Tf - Ti\_cont) = m\_Al \cdot c\_Al \cdot (Ti\_Al - Tf)$, or $1.5 \cdot 4200 \cdot (40 - 20) + 200 \cdot (40 - 20) = 0.900 \cdot c\_Al \cdot (200 - 40)$. Therefore $c\_Al = 900$ J/(K·kg).

P2. In the diagram showing the temperatures of water and copper as functions of time, draw the horizontal line indicating the final common temperature. Draw tangents to the temperature functions representing the rates of change of temperatures near the beginning. These tangents should intersect the common temperature at a common point. In this manner we find a time constant of about 30 s. The time constant must be equal to the product of the total (energy) capacitance and the energy transfer resistance from the water to the copper cylinder. The inverse of the total (energy) capacitance of the system is obtained from the sum of the inverse capacitances of water and copper; we obtain 830 J/K. This yields an energy transfer resistance of 0.040 K/W. This, in turn, is the re-

sult of the resistance of the convective interface and the resistance due to heat conduction through a part of the radius of the copper cylinder.

P3. $C_1 \cdot dT_1/dt = - (T_1 - T_2)/R$ and $C_2 \cdot dT_2/dt = - (T_1 - T_2)/R$. This is equivalent to $dT_1/dt = - (T_1 - T_2)/(R \cdot C_1)$ and $dT_2/dt = - (T_1 - T_2)/(R \cdot C_2)$. Subtracting the latter from the former yields $d(T_1 - T_2)/dt = - 1/R \cdot (1/C_1 + 1/C_2) \cdot (T_1 - T_2)$. This is a linear differential equation with constant coefficients for $T_1 - T_2$. Its solution is $T_1 - T_2 = (T_{10} - T_{20}) \cdot \exp(-t/\tau)$ where $\tau = R \cdot (1/C_1 + 1/C_2)^{-1}$ is the time constant. Obviously, $1/(1/C_1 + 1/C_2)$ is the total (energy) capacitance of the system of two bodies.

Part IV: THE DYNAMICS OF HEAT

MELTING AND VAPORIZATION

Assignment 1

Q1. Entropy.

Q2. The temperature stays constant. Entropy and amounts of substance of the phases change. The specific entropy of water is constant. The energy of the system of water and steam changes.

Q3. Laws of balance of entropy, amount of substance, and energy.

Q4. $dS/dt = IS\_heater - IS\_loss - IS\_steam$. IS_loss goes through the container wall, IS_steam is the convective current with the steam.

Q5. The specific entropy is constant. The total entropy changes (see Q4).

Q6. At constant temperature, the rate of entropy production is constant. If there is no variable entropy loss, the rate of production of steam should be constant. Amounts of water and steam should therefore decrease and increase linearly.

Q7. $Pi\_S = U \cdot I\_Q/T\_ev$.

Q8. There is hardly any loss (through the walls of the container). Therefore, the entropy produced is used solely to produce steam. In the case of energy, a part of what is added in heating is emitted to the environment in the expansion of steam.

Q9. Nearly so. In a uniform system (if it existed), the entropy added in vaporization is removed during condensation. No entropy is produced.

Q10. No, it is not. Entropy flows from the hot lava into the cold water. Here, entropy is produced.

Q11. In your own words…

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P1. From the graph of m(t): $dm/dt = -1.89e{-}4$ kg/s. $dn/dt = 1/Mo \cdot dm/dt = -1.05e{-}2$ mole/s.

P2. Boiling starts at around t = 60 s. Therefore, boiling lasts for 740 s – 60 s = 680 s. The amount of entropy generated is $S\_gen = 438/(273 + 98.1) \cdot 680$ J/K = 803 J/K. This entropy goes into the steam. The second question (about the water) does not have a unique answer. The most reasonable thing to say is that the specific entropy does not change.

Assignment 2

Q1. During fusion, entropy is emitted. During melting, the system takes up entropy.

Q2. The processes in the experiment are not reversible (heat transfer with temperature differences!). The melting itself can be approximated as being reversible.

Q3. We need to know the rate of production of water, and the latent entropy of fusion of ice.

Q4. They are proportional since the temperature of the process is constant. The factor of proportionality is the temperature of fusion. [$l_W = T_f \cdot l_S$; with $T_f = const$ we have $Q = T_f \cdot S_e$.]

Q5. Assume the power of the heater to be constant. The temperature should go up more or less linearly (actually, because of the increasing specific heat, the slope should decrease somewhat). Then, at around 0°C, there should be a plateau in the curve (not totally horizontal, though), after which the temperature rises again more or less as before.

Q6. In your own words…

P1. $S_e = S_{e\_water} + S_{e\_fusion} + S_{e\_ice} = m \cdot c_w \cdot \ln(293/273) + m \cdot lambda_S + m \cdot c_{ice} \cdot \ln(273/253) = 1680$ J/K. $W_e = 459$ kJ (which is roughly equal to 1680 J/K · 273 K).

P2. How much ice of 0°C is needed to make water of 8°C if it is mixed with 1 kg of water at 25°C? How much entropy is produced as the result of the process? Water and ice are inside a well insulated box. Balance of energy: $m_{ice} \cdot (q + c_w \cdot (8°C − 0°C)) = m_w \cdot c_w \cdot (25°C − 8°C)$, therefore $m_{ice} = 0.19$ kg. $S_{prod} = DS_{ice} – DS_{water} = 14.5$ J/K.

P3. Balance of energy: $m_{ice} \cdot (c_{ice} \cdot (0°C − (−12°C)) + q + c_w \cdot (T_{final} - 0°C)) + m_{tt} \cdot c_{tt} \cdot (T_{fine} − (−12°C)) = (m_w \cdot c_w + m_c \cdot c_c) \cdot (16.9°C − T_{final})$, therefore $T_{final} = 7.6°C$. Here, $m_{tt} = 0.086$ kg, $c_{tt} = 840$ J/(K·kg) have been taken for the test tube, and $m_c = 0.20$ kg and $c_c = 840$ J/(K·kg) for the glass container. According to the data, at $t = 6000$ s, the temperatures are finally equal at 9.6°C. The difference is due to the mixing of the water and the flow of entropy from the environment into the system.