

PART II Driving Forces, Flows, and Models

In the previous part we have produced some qualitative explanations of why fluid processes function the way they do. Basically, we believe that *level or pressure differences produce flows that lead to changes in stored quantities of liquids. In turn, the changes in stored quantities produce changes in pressures.* Here we will use this image and, with the help of some graphics oriented system dynamics software, turn the explanation into models that can be used to compute the consequences of the assumptions. The computed results are then compared to data. The models are quantitative and formal. Still, it is rather simple to create them from informal qualitative reasoning.

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II.1 A FIRST MODEL OF EQUILIBRATION

For almost 20 years now software has been available that maps the same idea we have used for explaining processes onto a graphical interface: *Physical processes are the result of the flow and the storage of certain quantities.* With liquids, we have perfect candidates to enter the world of dynamical modeling.

Let us start with the phenomenon of equilibration in two communicating tanks (see Section I.1, Fig. 3). To begin with, we take two equal cylindrical tanks filled with some oil. There is more oil in the left tank than in the right one. If the pipe is open, oil will flow from the left tank—where there is more oil—into the tank on the right (Fig. 39). This obvious statement can be expressed graphically in a system dynamics program by placing two storage symbols connected by a flow symbol (Fig. 40).

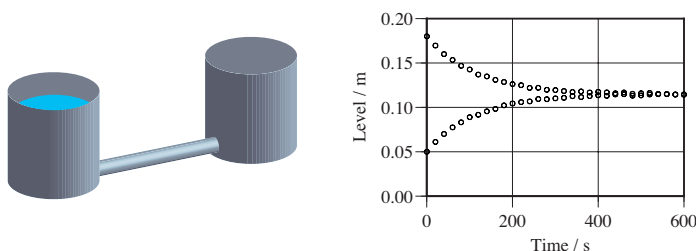


Figure 39: Observation of the levels of oil in two communicating tanks as in Fig. 3. In this example, both tanks are the same size. Note that the equilibrated levels are in the middle between the initial levels. Again, the levels change quickly at the beginning, and more slowly later on.

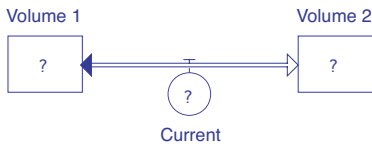


Figure 40: Representation of the idea that a fluid can be stored in two tanks (storage symbols on the left and on the right) and can flow from one tank into the other (pipeline symbol). The particular software used here is Stella (isee - Systems, Lebanon, NH). The idea expressed is called the law of balance of volume.

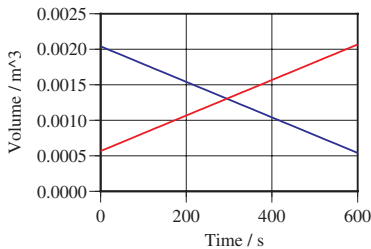


Figure 41: Simulation results of the model using a constant value for the current of oil.

Note that the storage symbols are labeled *Volume 1* and *Volume 2*, not Tank 1 and Tank 2. The point is that the symbols represent the quantities of liquid stored, not the storage tanks. We have chosen the volume of the oil to stand for its amount. The symbol called *Current* represents the current of volume of liquid.

When you compose a diagram such as this one in any of the common system dynamics programs, the software does something important. It creates an equation that takes care of the accounting of quantities of liquid in the tanks. If one tank loses an amount of fluid, the other one gains that amount. If one tank loses its liquid fast, the other one gains it equally fast. One can inspect the equation in the program, but we will not do so at this point. We will see such equations so often during our studies that we will know them by heart before long. It suffices to say that what we have expressed in graphical language are called *laws of balance*, here the laws of balance of volumes of fluid, for the two tanks connected by a pipe.

In Fig. 40, each of the three symbols carries a question mark. This means that we have to specify some quantities or relations to turn the diagram into a complete model that can be simulated. The storage symbols require us to assign the *initial value*, i.e., the starting value, of volume in the respective tank. Since the cross section of each tank is 0.01131 m^2 , and the measured initial values of the oil levels are 0.18 m on the left, and 0.050 m on the right, respectively, there are $0.01131 \cdot 0.18 \text{ m}^3 = 2.04 \cdot 10^{-3} \text{ m}^3$ of oil in the left tank, and $0.01131 \cdot 0.05 \text{ m}^3 = 5.66 \cdot 10^{-4} \text{ m}^3$ in the right one. Once these values have been specified and entered, the question marks for the storage symbols disappear.

The question mark in the pipeline symbol is not that easily removed, at least not if we want to create a useful model. But let us proceed in small steps. Assume that the flow of oil from the left to the right tank takes a certain constant value. If we enter such a value for the current, the model becomes complete, and it can be simulated. Simulation simply means solving the equations that represent the model. Here, the equations are those that do the accounting of amount of liquid for the two tanks, and the expression for the current, which is a constant.

If we set the current equal to $0.25 \cdot 10^{-5} \text{ m}^3/\text{s}$, the simulation results are those shown in Fig. 41. We could have performed this simulation in our heads. If the flow is constant, the volume on the left decreases at a constant rate, and the one on the right increases at the same constant rate.

Clearly the model is not useful. We know from experience that the current is not constant. It decreases as time goes on. So we need a better idea of how to calculate the flow of oil through the pipe. It does not appear to be far fetched to assume that the flow depends upon the difference of the amounts of fluid. (Actually, we will soon realize that the idea is not so good after all. For the moment, however, let us proceed.) To enter this idea into the model, we need to introduce the difference of volumes which can be done by calculating a new variable that depends upon Volume 1 and Volume 2 (Fig. 42). The simplest form of an expression for the current is obtained by saying that it is proportional to the volume difference:

$$I_V = k\Delta V \quad (1)$$

I_V is the symbol for the current of volume of oil, $\Delta V = V_1 - V_2$ stands for the difference of the two volumes, and k is a constant of proportionality called the flow factor. There is a simple interpretation for this constant. It tells us how easily the fluid flows. It depends on the specific setup of the system such as the properties of the fluid and the dimensions of the pipe. A larger value of k makes the flow larger, a smaller value makes it smaller. With this equation, the model is again complete.

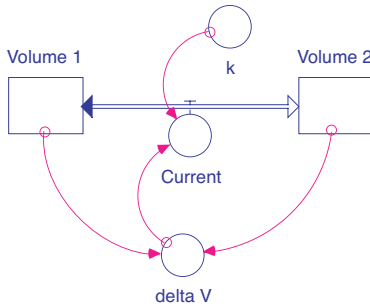


Figure 42: A model where the flow depends upon the difference of the amounts of fluid stored in the two tanks. Since the difference decreases, the flow is expected to decrease as well.

II.2 SIMULATION AND COMPARISON TO OBSERVATIONS

A complete model can be simulated, i.e., the model equations can be solved. The result is a set of functions of time that represent the consequences of the assumptions that went into the model. Once we know the consequences of our thinking, we can compare them to data obtained from observations. If the calculations do not fit the data, we can try to make the simulation fit better by changing parameters of the model. In our example, the first relevant parameter is the flow factor k in Equ. 1.

The solution of the model equations by the software proceeds as follows. Since the initial values of the volumes (Fig. 42) are known, the volume difference can be calculated and from this, with the help of the assumed value of k , the current is obtained. Now the software uses the value of the current and calculates (for a very small time step) by how much the volumes on the left and on the right will change. With these new values of V_1 and V_2 , the computation is repeated. Step by step, the simulated values of V_1 and V_2 are obtained.

Since we know the cross sections of the two tanks in our experiment, we can calculate the oil levels from the volumes. Now we are ready to compare simulation results and experimental data (Fig. 39). At first, having used an arbitrary value for k , the two are not expected to agree. However, if we try out different values for k we may be able to get better results. Indeed, for $k = 0.0050$, we get an almost perfect agreement as shown in Fig. 43.

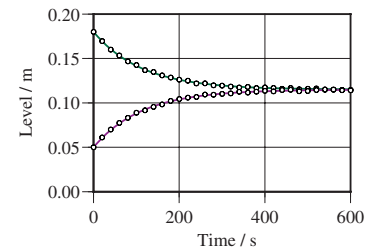


Figure 43: Data (circles) and simulation (solid lines) for our experiment and the model shown in Fig. 42.

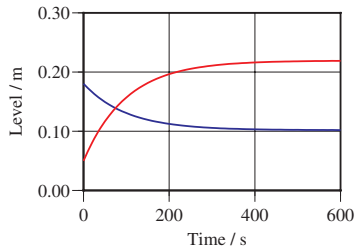


Figure 44: Predicted oil levels if the cross sections of the two tanks are not equal.

This exercise demonstrates what modeling is good for. It tests the quality of our ideas (it does not prove them; there is no proof), and it allows us to determine unknown parameters that are part of the expressions of our ideas, such as the flow factor k in Equ. 1.

We have a model, and it works well for the one observation which has been reported in Fig. 39. Now we have to perform an important step. We should use the model to predict the behavior of similar but not necessarily identical systems. The prediction of these simulation runs should again be compared with reality for which we may have to perform some additional observations. Since we already have data for the case of tanks with different cross sections (Fig. 3) we can use the model on this example. The result, which is shown in Fig. 44, probably does not surprise you: it does not work at all, it is clearly wrong. Our model predicts differing oil levels for systems having tanks with unequal cross sections!

II.3 A SECOND MODEL OF EQUILIBRATION

Something is wrong with the model, and we know what: It assumes that quantities equilibrate instead of levels of fluid. Check out Equ. 1. It tells us that the flow stops when amounts of liquid on the left and on the right have become equal. In our discussion in Part I we have realized that nature does not appear to work that way. The simplest idea that can explain what we routinely see is this: Flows in systems of communicating tanks depend upon level differences. (We know from the observation reported in Fig. 4 that not even this is always true. Rather, we should *take the pressure difference as the driving force of a flow*. But for now let us continue with level differences.) This means that we should change the relation for the current to something like this:

$$I_V = k\Delta h \quad (2)$$

Here, $\Delta h = h_1 - h_2$ is the difference of levels of liquid on the two sides of the system. The equation informs us that the flow will double if the level difference is doubled. Again k is a flow factor that tells us how easily the oil flows through the given pipe. However, it is not exactly the same quantity as the factor in Equ. 1 since the relations in the equations are expressed with the help of different variables.

The model in Fig. 42 has to be changed slightly. Since we need the level difference to express the current, we first have to calculate the levels of oil from their volumes for which we need the cross sections of the tanks (Fig. 45). Using the levels, the level difference is calculated. Finally, the assumed form of the relation between level difference and current, i.e., Equ. 2, is entered into the model.

Now we are ready to simulate the model and compare the results and the data reported before. The comparison is presented in the level-time graphs of Fig. 45. We see that model and experiment yield closely matching results. We are satisfied and

say that the idea expressed in Equ. 2 is successful. As before, simulation results and data are used to find a proper value of the flow factor. In our example, it is equal to $0.50 \cdot 10^{-4} \text{ m}^2/\text{s}$.

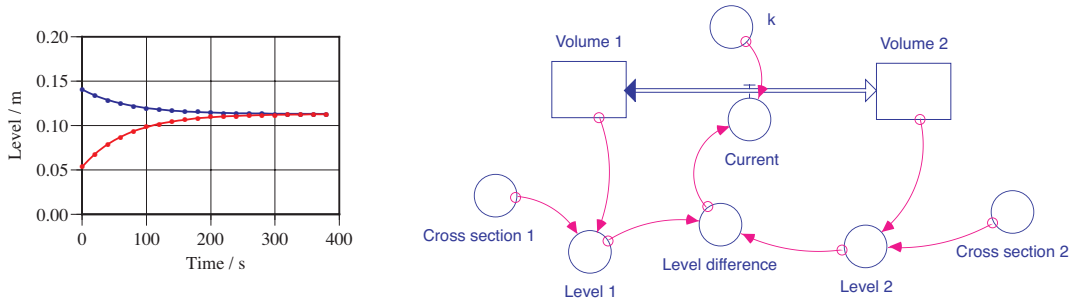


Figure 45: The revised model that uses level differences as the driving force of flows, and the comparison of simulation results (solid lines in the graph) and data (bullets).

II.4 WHEN WATER FLOWS

Next we will change the system once more. Instead of oil we use water. Data is taken, and the model created for the previous case is used for a simulation. We adjust the value of the flow factor k so that the fit between data and simulation becomes as good as possible. We expect it to be larger than for oil if everything else about the system (the pipe) is the same. The result for the best possible fit is shown in Fig. 46. We see that it is not nearly as good as what we could have expected from previous examples. The value of k is not at fault here. If we take a close look at the functions we see that the shape of the simulated curves is different from the measured ones. This indicates a deeper reason for the difference between model and experiment. What is different? The model contains three relations. First there is the law of balance of amount of liquid (Fig. 40). This is not affected by the change. Neither is the calculation of levels from cross sections since the tanks all have the same shape. The only possible difference is the flow law we have used so far, i.e., the expression in Equ. 2. This has to be reconsidered.

Here is how we can produce new ideas. We ask ourselves two related questions:

- *What quantities does the current depend upon?*
- *How does it depend upon these quantities?*

The answer to the first question is the same as before. The flow depends upon the level difference and some quantities that define the properties of the system such as fluid properties and dimensions of the pipe. Of all these factors, the level difference

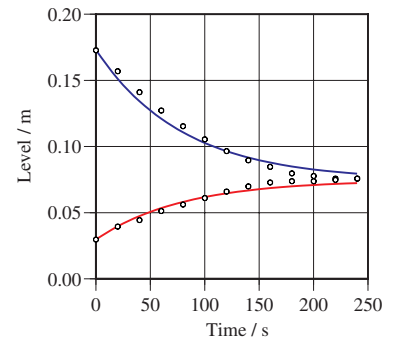


Figure 46: Simulation results (solid lines) and experimental data (circles) for the flow of water in a system of two communicating tanks.

is the only one that changes with time. All the others are constant, at least for a given situation. Therefore, the latter ones can be combined into a single factor that influences the current for a fixed experimental setup.

The only difference in our model can come from a different answer to the second question: How does the current depend upon the level difference? The previous answer was the simplest possible one: Double the level difference yields double the flow. Stated differently, we assumed a direct proportionality between level difference and current, i.e., we assumed a linear relation to be valid.

We know one thing about the relation for certain: A larger level difference should lead to a larger flow. It is possible to satisfy this condition in many different ways. The basic alternatives are these: The current can grow faster than a linear function with growing level difference, or it can grow more slowly. For the first alternative we could arbitrarily assume a simple mathematical expression such as a square function:

$$I_V = k(h_1 - h_2)^2 \quad (3)$$

The second could be satisfied by taking the square root of the level difference:

$$I_V = k\sqrt{h_1 - h_2} \quad (4)$$

The alternatives are shown graphically in Fig. 47. Maybe you have a feeling for which of these possibilities appears to be more reasonable. If not, both can be implemented in our model, and the results can be compared to the data of the experiment. It turns out that with Equ. 3 things get visibly worse. There is no chance for a good fit. However, the idea of the square root function Equ. 4 works well. We obtain a good correspondence between data and simulation (Fig. 48).

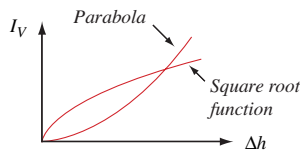


Figure 47: Parabola and square root functions compared.

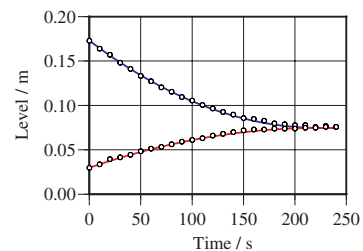


Figure 48: Data of the flow of water in a system of communicating tanks (circles) and simulation results (solid lines) for a flow model as in Equ. 4.

Compare our results to the discussion of laminar and turbulent flow characteristics (see Fig. 12). Clearly, the successful models for the flow of oil or water correspond to these two cases, respectively.

II.5 TWO TANKS AND AN OUTFLOW

Finally, let us make the system a little more complex by adding an outflow to the larger of two communicating tanks (Fig. 22). The tanks contain water, and the smaller of the two tanks has an initial fluid level that is smaller than the one in the large tank.

Our model starts with the one produced so far, i.e., the one corresponding to the diagram of Fig. 45, with a flow law taken from Equ. 4. Since there is a second pipe connected to the larger tank (Tank 2), we simply add a symbol for this flow to the previous model (Fig. 49). This flow is assumed to depend upon the difference of fluid levels in Tank 2 and the outlet (“ambient” level), and upon a flow factor which is different from the first flow factor if the second pipe is different. Therefore we have two unknown parameters in this model that have to be determined independently by fitting simulation results to data taken in an experiment (graph in Fig. 49). The result for this extended example is clearly satisfactory.

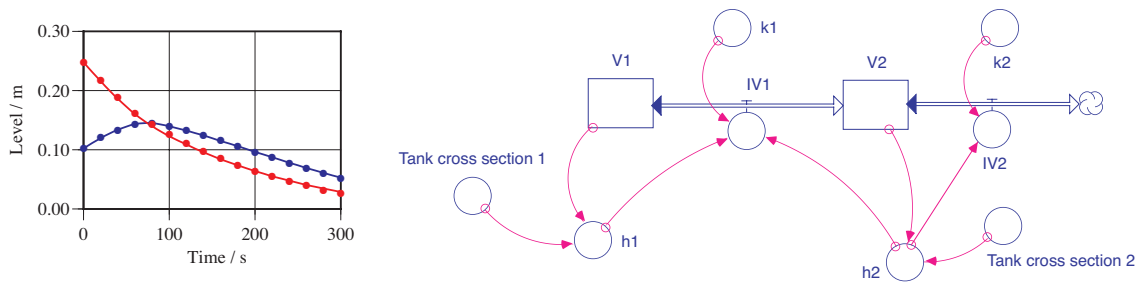


Figure 49: The revised model that uses level differences as the driving force of flows (right), and the comparison of simulation results (solid lines in the graph) and data (bullets, left).

II.6 MODELING AND THE SCIENTIFIC METHOD

Dynamical modeling is not only simple, it is a great tool for testing ideas concerning physical processes. System dynamics models use a kind of reasoning that takes our images of fluids and turns them into representations that can be simulated and compared to data taken from nature and experiments. If the test turns out negative, it is easy to change the models and try out new ideas.

These introductory examples tell us something rather important about the scientific method. We have some experience with certain phenomena. Somehow we develop a hunch about why things run the way they do, and we use this hunch to formulate an idea or hypothesis. This hypothesis is turned into a formal model, the model is

simulated, and the results are compared to data taken in the lab. If we like the comparison, we call the hypothesis successful. Note that there is no proof whatsoever that nature is “really like” what we assume in our hypothesis. The process of arriving at a theory is indirect: It does not go directly from observation to a truth formulated in laws.